

2016 年

普陀区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共 6 题,每题 4 分,满分 24 分)

1. B 2. C 3. A 4. D 5. B 6. C

二、填空题:(本大题共 12 题,每题 4 分,满分 48 分)

7. $m(a+b)(a-b)$ 8. $x=2$ 9. $-1 < x < 2$ 10. 2 11. $x \neq 0$ 12. 2400 13. $\frac{1}{3}$ 14. $\frac{1}{2}b - \frac{1}{2}a$

15. 22 16. $<$ 17. $\frac{3}{8}$ 18. $(\frac{3}{2}, 2)$

三、解答题

(本大题共 7 题,其中第 19—22 题每题 10 分,第 23、24 题每题 12 分,第 25 题 14 分,满分 78 分)

19. 解:原式 $= -9 + 2 - \sqrt{3} + 9 - \sqrt{3} - 1$ (8 分)

$= 1 - 2\sqrt{3}$. (2 分)

20. 解:方程②可变形为 $(x-y)(x-2y)=0$. (1 分)

得: $x-y=0$ 或 $x-2y=0$, (2 分)

原方程组可化为 $\begin{cases} x^2 + y^2 = 5, \\ x - y = 0; \end{cases} \begin{cases} x^2 + y^2 = 5, \\ x - 2y = 0. \end{cases}$ (2 分)

解得: $\begin{cases} x_1 = \frac{1}{2}\sqrt{10}, \\ y_1 = \frac{1}{2}\sqrt{10}; \end{cases} \begin{cases} x_2 = -\frac{1}{2}\sqrt{10}, \\ y_2 = -\frac{1}{2}\sqrt{10}; \end{cases} \begin{cases} x_3 = 2, \\ y_3 = 1; \end{cases} \begin{cases} x_4 = -2, \\ y_4 = -1. \end{cases}$ (4 分)

$\therefore$ 原方程组的解是 $\begin{cases} x_1 = \frac{1}{2}\sqrt{10}, \\ y_1 = \frac{1}{2}\sqrt{10}; \end{cases} \begin{cases} x_2 = -\frac{1}{2}\sqrt{10}, \\ y_2 = -\frac{1}{2}\sqrt{10}; \end{cases} \begin{cases} x_3 = 2, \\ y_3 = 1; \end{cases} \begin{cases} x_4 = -2, \\ y_4 = -1. \end{cases}$ (1 分)

21. 解:过点 A 作 $AE \perp BC$, 垂足为点 E. (1 分)

$\because AP^2 = AD \cdot AB, AB = AC,$

$\therefore AP^2 = AD \cdot AC$. (1 分)

$\therefore \frac{AD}{AP} = \frac{AP}{AC},$

$\angle PAD = \angle CAP,$ (1 分)

$\therefore \triangle APD \sim \triangle ACP$. (1 分)

得 $\angle APD = \angle C$. (1 分)

$\because AB = AC, AE \perp BC, \therefore CE = \frac{1}{2}BC = 12$. (2 分)

$\because AE \perp BC, AC = 13, \therefore$ 由勾股定理得 $AE = 5$. (1 分)

$\therefore \sin C = \frac{AE}{AC} = \frac{5}{13}$. (1 分)

即 $\sin \angle APD = \frac{5}{13}$. (1 分)

22. 解: 设李师傅的平均速度为 x 千米/时, 王师傅的平均速度为 $(x-20)$ 千米/时. (1分)

根据题意, 可列方程 $\frac{200}{x-20} - \frac{200}{x} = \frac{1}{2}$. (3分)

整理得 $x^2 - 20x - 8000 = 0$.

解得 $x_1 = 100, x_2 = -80$. (2分)

经检验, $x_1 = 100, x_2 = -80$ 都是原方程的解. 因为速度不能负数, 所以取 $x = 100$. (1分)

李师傅的最快速度是: $100 \times (1 + 15\%) = 115$ 千米/时, 小于 120 千米/时. (2分)

答: 李师傅没有超速. (1分)

23. 证明: (1) $\because AD \parallel BC, DF \parallel AB$,

$\therefore$ 四边形 $ABFD$ 是平行四边形. (1分)

$\because AD \parallel BC, \therefore \angle ADB = \angle DBC$. (1分)

$\because BD$ 平分 $\angle ABC, \therefore \angle ABD = \angle DBC$. (1分)

$\therefore \angle ABD = \angle ADB$. (1分)

$\therefore AD = AB$. (1分)

$\therefore$ 四边形 $ABFD$ 是菱形. (1分)

(2) 联结 OF .

$\because AC \perp AB, \therefore \angle BAO = 90^\circ$.

$\because$ 四边形 $ABFD$ 是菱形, $\therefore AB = BF$. (1分)

又 $\because \angle ABO = \angle OBF, BO$ 是公共边, $\therefore \triangle ABO \cong \triangle FBO$.

$\therefore \angle BFO = \angle BAO = 90^\circ$. (1分)

$\because DF \parallel AB, \therefore \angle FEC = \angle BAO = 90^\circ$. (1分)

$\because \angle EFC + \angle ECF = 90^\circ, \angle EFC + \angle OFE = 90^\circ$,

$\therefore \angle OFE = \angle ECF$. (1分)

又 $\because \angle BAC = \angle FEO, \therefore \triangle ABC \sim \triangle EOF$. (1分)

$\therefore \frac{AB}{OE} = \frac{AC}{EF}$. (1分)

即: $AC \cdot OE = AB \cdot EF$.

24. (1) 解: 把 $x = 4$ 代入 $y = \frac{8}{x}$, 得 $y = 2$.

$\therefore$ 点 B 的坐标为 $(4, 2)$. (1分)

$\because$ 直线 AC 的截距是 $-6, \therefore$ 点 A 的坐标为 $(0, -6)$. (1分)

$\because$ 二次函数的 $y = \frac{1}{3}x^2 + bx + c$ 的图像经过点 A, B ,

$\therefore$ 可得: $\begin{cases} \frac{1}{3} \times 16 + 4b + c = 2 \\ c = -6 \end{cases}$, 解得: $\begin{cases} b = \frac{2}{3} \\ c = -6 \end{cases}$.

$\therefore$ 二次函数的解析式是 $y = \frac{1}{3}x^2 + \frac{2}{3}x - 6$. (2分)

(2) $\because BC \parallel x$ 轴,

∴点C的纵坐标为2.

把 $y=2$ 代入 $y=\frac{1}{3}x^2+\frac{2}{3}x-6$, 解得 $x=4, x=-6$.

∵(4,2)是点B的坐标, ∴点C的坐标为(-6,2).

(2分)

设直线AC的表达式是 $y=kx-6$,

∵点C在直线AC上, ∴ $k=-\frac{4}{3}$.

∴直线AC的表达式是 $y=-\frac{4}{3}x-6$.

(1分)

(3) ① $BC \parallel AD_1$

设点 D_1 的坐标是 $(m, -6)$,

由 $D_1C=AB$, 可得: $(6+m)^2+64=16+64$,

解得: $m=-2, m=-10$ (舍).

∴点 D_1 的坐标是 $(-2, -6)$.

(2分)

② $AC \parallel BD_2$

可得: 直线 BD_2 的表达式是 $y=-\frac{4}{3}x+\frac{22}{3}$.

设点 D_2 的坐标是 $(n, -\frac{4}{3}n+\frac{22}{3})$,

由 $AD_2=BC$, 可得: $n^2+(-\frac{4}{3}n+\frac{22}{3}+6)^2=100$,

解得: $n=\frac{14}{5}, n=10$ (舍).

∴点 D_2 的坐标是 $(\frac{14}{5}, \frac{18}{5})$.

(2分)

③ ∵ $AC=BC$,

∴ $CD_3 \parallel AB$ 不存在.

(1分)

综上所述, 点D的坐标是 $(-2, -6)$ 或 $(\frac{14}{5}, \frac{18}{5})$.

25. (1) 解: 作图正确.

(2分)

设AD的垂直平分线与AB交于点E, 垂足是点H.

在 $Rt\triangle AHE$ 中, 由 $\tan A = \frac{3}{4}$, $AD=8$, 得: $AE=5, EH=3$.

所以圆E的半径长等于5.

(2分)

(2) ∵ $\angle 1 + \angle C = \angle 2 + \angle EFG$, $\angle C = \angle EFG = 90^\circ$, ∴ $\angle 1 = \angle 2$.

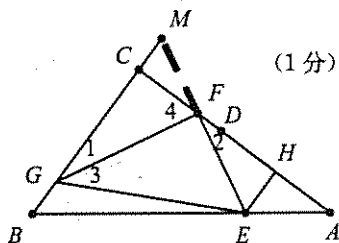
又 ∵ $\angle C = \angle DHE = 90^\circ$,

∴ $\triangle CFG \sim \triangle HEF$.

∴ $\frac{HE}{CF} = \frac{FH}{CG}$. ∴ $\frac{3}{14-x} = \frac{x-4}{y}$.

化简得: $y = \frac{-x^2+18x-56}{3}$ ($4 \leq x < 14$). (2分+1分)

(3) ① 当点G在边BC上时



(1分)

$\triangle EFG$ 与 $\triangle FCG$ 相似, 有两种可能.

当 $\angle 3 = \angle 4$ 时, 可得: $CF \parallel EG$.

易证四边形 $HCGE$ 是平行四边形.

$$\therefore y = EH = 3, EG = HC = 10.$$

$$\therefore r_G + r_E = 8 < 10,$$

$\therefore$ 两圆外离.

(2 分)

当 $\angle 1 = \angle 3$ 时, 延长 EF 与 BC 的延长线相交于点 M ,

可证得 $MF = EF$, 由 $\triangle MCF \cong \triangle EHD$, 可得: 点 F 是 CH 的中点.

$$\therefore HF = 5, y = \frac{25}{3}, EG = MG = \frac{34}{3}.$$

$$\therefore r_G + r_E = \frac{40}{3}, r_G - r_E = \frac{10}{3},$$

$\therefore$ 两圆相交.

(2 分)

② 当点 G 在 BC 延长线上时

$\triangle EFG$ 与 $\triangle FCG$ 相似, 只能是 $\angle 1 = \angle 2$.

设 EG 与 AC 交于点 N ,

易证: 点 N 是 EG 的中点.

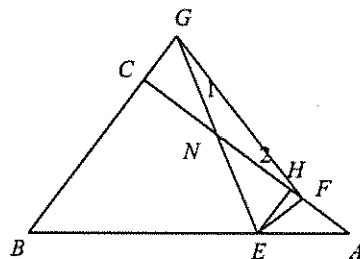
由 $\triangle CNG \cong \triangle HNE$,

可得 $CG = 3, EG = 2\sqrt{34}$.

$$\therefore r_G + r_E = 8 < 2\sqrt{34},$$

$\therefore$ 两圆外离.

(2 分)



奉贤区中考数学质量抽查试卷 · 参考答案

一、选择题: (本大题共 6 题, 每题 4 分, 满分 24 分)

1. C 2. B 3. C 4. B 5. A 6. D

二、填空题: (本大题共 12 题, 每题 4 分, 满分 48 分)

7. $4\sqrt{a}$ 8. $a(a-1)$ 9. $x \neq 1$ 10. 1 11. $x > 3$ 12. 减小 13. $y = \frac{1}{2}x + 2$ 14. $6\sqrt{3}$

15. $\frac{2}{3}\vec{a} - \frac{1}{2}\vec{b}$ 16. $AD = BC$ 等(答案不唯一) 17. $\frac{\sqrt{3}}{2}$ 18. $\sqrt{3} + 1$

三、解答题: (本大题共 7 题, 满分 78 分)

19. (本题满分 10 分)

$$\begin{aligned} \text{解: 原式} &= 1 - \frac{\sqrt{2}}{2} - 2 + 2 - \frac{\sqrt{2}}{2} \\ &= 1 - \sqrt{2} \end{aligned}$$

各 2 分

2 分

20. (本题满分 10 分)

解: 方程两边同乘以 $(x^2 - 4)$

1 分

$$\text{得: } (x+2)^2 - (x-2) = 16$$

3 分

整理,得: $x^2 + 3x - 10 = 0$

解得: $x_1 = 2, x_2 = -5$

经检验: $x_1 = 2$ 是增根, $x_2 = -5$ 是原方程的根

所以原方程的根是 $x = -5$

21. (本题满分 10 分, 每小题满分各 5 分)

解:(1) $\because AB = 4, \frac{BE}{AB} = \frac{1}{4} \therefore BE = 1$

$\because DE \perp AD, \angle ACB = 90^\circ \therefore \angle CAD + \angle ADC = \angle BDE + \angle ADC \therefore \angle CAD = \angle BDE$

$\because AD$ 是 $\angle BAC$ 的角平分线, $\therefore \angle CAD = \angle BAD \therefore \angle BAD = \angle BDE$

$\therefore \angle B = \angle B \therefore \triangle BDE \sim \triangle BAD$

$\therefore \frac{BE}{BD} = \frac{BD}{AB} \therefore BD = 2$

(2) 过点 D 作 $DH \perp AB$ 于点 H

$\therefore \angle AHD = 90^\circ \because AD$ 是 $\angle BAC$ 的角平分线, $\angle ACB = 90^\circ \therefore CD = DH$

$\because \angle AHD = \angle ACB = 90^\circ, \angle B = \angle B, \triangle BDH \sim \triangle BAC$

$\therefore \frac{DH}{AC} = \frac{BD}{AB} = \frac{2}{4} = \frac{1}{2}, \therefore \frac{CD}{AC} = \frac{1}{2}$

$\therefore$ 在 $\text{Rt}\triangle ACD$ 中, $\angle ACD = 90^\circ, \tan \angle ADC = \frac{AC}{CD} = 2$ 即: $\angle ADC$ 的正切值为 2

22. (本题满分 10 分, 第(1)小题 4 分, 第(2)小题 6 分)

(1) 50, 60;

(2) 设参与敬老院服务的六、七年级学生分别有 x 人、 y 人

根据题意, 得:
$$\begin{cases} x + y = 60 \\ (1 + 40\%)x + (1 + 60\%)y = 90 \end{cases}$$

解得
$$\begin{cases} x = 30 \\ y = 30 \end{cases}$$

答: 参与敬老院服务的六、七年级学生各有 30 人.

23. (本题满分 12 分, 每小题满分各 6 分)

证明:(1) $\because DC \parallel AB, AD = BC = DC$

$\therefore \angle DCB = \angle ADC, \angle DCB = \angle CBE \therefore \angle ADC = \angle CBE$

$\because \angle BCE = \angle ACD, BC = DC \therefore \triangle ADC \cong \triangle EBC$

$\therefore AD = BE \therefore DC = BE$

$\because DC \parallel AB \therefore$ 四边形 $DBEC$ 是平行四边形

(2) $\because$ 四边形 $DBEC$ 是平行四边形 $\therefore BD = CE$

$\because DC \parallel AB, AD = BC = DC \therefore AC = BD \therefore AC = BD$

$\because \angle DCA = \angle CAB \angle BCE = \angle ACD \therefore \angle BCE = \angle CAB$

$\therefore \angle E = \angle E \therefore \triangle ECB \sim \triangle EAC$

$\therefore \frac{BE}{EC} = \frac{EC}{AE} \therefore CE^2 = BE \cdot AE$ 即 $AC^2 = AD \cdot AE$

24. (本题满分 12 分, 每小题满分各 4 分)

(1) $\because$ 抛物线 $y = -x^2 + bx + c$ 交 x 轴交于点 $A(-1, 0)$ 和点 $C(3, 0)$

$$\therefore \begin{cases} -1 - b + c = 0 \\ -9 + 3b + c = 0 \end{cases} \text{ 解得: } \begin{cases} b = 2 \\ c = 3 \end{cases}$$

3 分

$\therefore$ 该抛物线的解析式: $y = -x^2 + 2x + 3$

1 分

(2) 由 $y = -x^2 + 2x + 3$ 得点 $B(0, 3)$

1 分

$\because AD \perp CD \therefore \angle DBP + \angle BPD = 90^\circ \because \angle POA = 90^\circ \therefore \angle OAP + \angle APO = 90^\circ$

$\therefore \angle BPD = \angle APO \therefore \angle DBP = \angle OAP \therefore \angle AOP = \angle BOE = 90^\circ$

$\therefore \triangle AOP \sim \triangle BOE$

1 分

$$\therefore \frac{AO}{BO} = \frac{PO}{OE} \because OA = 1, PO = \frac{2}{3}, BO = 3 \therefore \frac{1}{3} = \frac{\frac{2}{3}}{OE} \therefore OE = 2$$

1 分

$$\because OC = 3 \therefore EC = 1 \therefore S_{\triangle EBC} = \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$$

1 分

(3) 设点 $P(0, y)$, 则 $OP = y, BP = 3 - y, AP = \sqrt{1 + y^2}$

$\because$ 点 D 在抛物线的对称轴上, 过点 D 作 $DH \perp x$ 轴, 垂足为点 H

$$\therefore AH = 2 \therefore AO = OH \therefore PD = AP = \sqrt{1 + y^2}$$

$\therefore \angle BPD = \angle APO \angle AOP = \angle BDP = 90^\circ \therefore \triangle AOP \sim \triangle BDP$

1 分

$$\therefore \frac{AP}{BP} = \frac{PO}{PD} \therefore \frac{\sqrt{1 + y^2}}{3 - y} = \frac{y}{\sqrt{1 + y^2}} \text{ 解得: } y_1 = 1, y_2 = \frac{1}{2}.$$

1 分

经检验: $y_1 = 1, y_2 = \frac{1}{2}$ 都是原方程的根 $\therefore P_1(0, 1), P_2(0, \frac{1}{2})$

2 分

25. (本题满分 14 分, 第(1)小题 5 分, 第(2)小题 5 分, 第(3)小题 4 分)

(1) 解: 当点 E 与点 D 重合时, $AE = 5, EF \parallel AB \therefore \angle ADF = \angle DAB$

1 分

过点 A 作 $AH \perp EF$ 于点 H

1 分

$\therefore$ 在 $\odot A$ 中, $EF = 2EH, \angle AHE = 90^\circ$

1 分

$$\therefore \cos \angle ADF = \cos \angle DAB = \frac{EH}{AE} = \frac{3}{5} \therefore EH = 3 \quad EF = 6$$

2 分

(2) 解: 过点 C 作 $CM \perp AD$ 交 AD 延长线于点 M

1 分

在 $Rt\triangle CMD$ 中, $\angle CMD = 90^\circ, \cos \angle MDC = \cos A = \frac{3}{5}, CD = 5$

$$\therefore MD = 3, \therefore CM = 4$$

1 分

在 $Rt\triangle CME$ 中, $\angle CME = 90^\circ, \therefore CE^2 = CM^2 + ME^2$

$$\because CM = 4, MD = 3, DE = 5 - x, CE = y \therefore y^2 = 4^2 + (3 + 5 - x)^2$$

1 分

$$\therefore y = \sqrt{x^2 - 16x + 80} (0 < x \leq 5)$$

2 分

(3) 解: 假设存在一点 P , 使得 $\widehat{EF} = 2\widehat{PE}$

过圆心 A 作 $AH \perp EF$ 于点 H , 交 $\odot A$ 为点 N

1 分

$$\therefore \widehat{EF} = 2 \cdot \widehat{EN}, \therefore \widehat{EF} = 2 \cdot \widehat{PE}, \therefore \widehat{PE} = \widehat{EN} \therefore \angle NAE = \angle PAE$$

1 分

$\because AH \perp EF, \therefore \angle NAE + \angle HEA = 90^\circ. \therefore \angle CME = 90^\circ, \therefore \angle CEM + \angle ECM = 90^\circ.$

$$\because \angle HEA = \angle CEM, \therefore \angle NAE = \angle ECM = \angle PAE = \angle MDC.$$

$$\therefore \tan \angle ECM = \tan \angle MDC = \frac{4}{3}$$

$$\therefore \text{在 Rt}\triangle CME \text{ 中, } \angle CME = 90^\circ, CM = 4, ME = MD + DE = 3 + 5 - x$$

$$\tan \angle ECM = \frac{ME}{MC} = \frac{8-x}{4} = \frac{4}{3}, \text{解得 } x = \frac{8}{3} \quad 2 \text{ 分}$$

即:存在点 P , 使得 $\widehat{EF} = 2 \cdot \widehat{PE}$, 此时 AP 长为 $\frac{8}{3}$.

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一、选择题

1. C 2. A 3. D 4. B 5. D 6. C

二、填空题

7. -1 8. $\sqrt{a}+b$ 等 9. $m=4$ 10. $x \neq 2$ 11. 4 12. $\frac{1}{4}$ 13. $-\frac{1}{3}\vec{a} + \frac{2}{3}\vec{b}$ 14. 2.4 15. 0.05

16. $y = \frac{2}{x}$ 等 17. $\frac{6}{5}$ 18. $\frac{\sqrt{2}}{2}$

三、解答题

$$19. \text{解: 原式} = 1 + 3 + 6 \times \frac{\sqrt{3}}{2} - 2\sqrt{3} \quad (8 \text{ 分})$$

$$= 4 + \sqrt{3}. \quad (2 \text{ 分})$$

$$20. \text{解: 由 } 2x - 1 > 3(x - 1), \text{ 得 } x < 2. \quad (3 \text{ 分})$$

$$\text{由 } \frac{5-x}{2} < x + 5, \text{ 得 } x > -\frac{5}{3}. \quad (3 \text{ 分})$$

$$\text{所以不等式组的解集为 } -\frac{5}{3} < x < 2. \quad (2 \text{ 分})$$

$$\text{其非负整数解为 } 0 \text{ 和 } 1. \quad (2 \text{ 分})$$

21. (1) 证明: $\because \angle ACB = 90^\circ, N$ 为 AB 的中点, D 为 BN 中点,

$$\therefore CN = \frac{1}{2}AB, CD = \frac{1}{2}BM. \quad (1 \text{ 分}, 1 \text{ 分})$$

$$\therefore \frac{CN}{AB} = \frac{CD}{BM} = \frac{1}{2}, \text{ 即 } \frac{CN}{AB} = \frac{CD}{BM} \quad (1 \text{ 分})$$

$$(2) \text{解: 联结 } ND, \because N, D \text{ 分别为 } AB, BM \text{ 的中点}, \therefore ND = \frac{1}{2}AM. \quad (1 \text{ 分})$$

$$\therefore \frac{CN}{AB} = \frac{CD}{MB} = \frac{1}{2}, \therefore \frac{CN}{AB} = \frac{CD}{MB} = \frac{ND}{AM} \therefore \triangle CND \sim \triangle BAM. \quad (1 \text{ 分})$$

$$\therefore \angle NCD = \angle ABM. \quad (1 \text{ 分})$$

$$\text{作 } MH \perp AB \text{ 于 } H, \because \angle A = 30^\circ, \therefore \text{设 } MH = k, \text{ 则 } AM = 2k, AH = \sqrt{3}k. \quad (1 \text{ 分})$$

$$\therefore AC = 2AM = 4k, AB = \frac{AC}{\cos A} = \frac{8\sqrt{3}}{3}k \quad (1 \text{ 分})$$

$$\therefore BH = AB - AH = \frac{8\sqrt{3}}{3}k - \sqrt{3}k = \frac{5\sqrt{3}}{3}k. \quad (1 \text{ 分})$$

$$\therefore \cot \angle ABM = \frac{BH}{MH} = \frac{5\sqrt{3}}{3}. \quad (1 \text{ 分})$$

22. 解: (1) 设 y 关于 x 函数解析式为 $y = kx (k \neq 0)$, (1 分)

则 $600 = 20k$, 得 $k = 30$. (1 分)

所以 $y = 30x (0 < x \leq 20)$. (2 分)

(2) 因为前 18 分钟内的平均速度与后 8 分钟内的平均速度之比为 2:3,

所以设前 18 分钟内的平均速度为 $2v$, 后 8 分钟内的平均速度 $3v$. (1 分)

则 $2v \times 18 + 3v \times 8 = 600$. (2 分)

解得 $v = 10$. (1 分)

所以, 前 18 分钟内的平均速度为 20 米/分, 行走的路程 = 360 米. (1 分)

所以, 点 C 的纵坐标为 240. (1 分)

23. 证明: (1) $\because DC \parallel AB, \therefore \angle A + \angle ADE = 180^\circ$. (1 分)

$\because \angle A = 90^\circ, \therefore \angle ADE = 90^\circ$. (1 分)

$\because$ 纸片沿过点 D 的直线翻折, 点 A 落在边 CD 上的点 E 处, 折痕为 DF,

$\therefore \triangle ADF \cong \triangle EDF, \therefore \angle A = \angle DEF = 90^\circ$, 且 $AD = ED$. (1 分, 1 分)

$\therefore$ 四边形 ADEF 为矩形. (1 分)

$\because AD = ED, \therefore$ 矩形 ADEF 为正方形. (1 分)

(2) 联结 DG, $\because DC \parallel AB, BG = CD, \therefore$ 四边形 DCBG 为平行四边形.

$\therefore CB = DG$. (1 分)

$\because$ 四边形 ADEF 为正方形, $\therefore \angle A = \angle EFA = 90^\circ, AD = EF$.

$\because AG = GF, \therefore \triangle DAG \cong \triangle EFG, \therefore DG = EG$.

$\therefore CB = EG$. (3 分)

$\because BG = CD = CE + DE, \therefore BG > CE$, 即 $BG \neq CE$.

$\because DC \parallel AB, \therefore$ 四边形 GBCE 为梯形. (1 分)

$\because CB = EG, \therefore$ 梯形 GBCE 为等腰梯形. (1 分)

24. 解: (1) $\because y = ax^2 - 8ax + 3$ 的对称轴是直线 $x = 4$, (1 分)

$\therefore B(4, 0), A(0, 3), \therefore AB = 5$. (1 分)

$\because AB = BD$, 且 $a < 0, \therefore D(4, 5)$. (1 分)

将点 D 的坐标代入 $y = ax^2 - 8ax + 3$, 解得 $a = -\frac{1}{8}$.

$\therefore$ 抛物线的表达式是 $y = -\frac{1}{8}x^2 + x + 3$. (1 分)

(2) 过点 P 作 $PH \perp BD$ 于点 H, $\because DP \parallel AB, \therefore \angle BDP = \angle ABD$.

$\because BD \parallel y$ 轴, $\therefore \angle OAB = \angle ABD, \therefore \angle BDP = \angle OAB$. (1 分)

$\therefore \frac{PH}{DH} = \tan \angle BDP = \tan \angle OAB = \frac{4}{3}$.

$\therefore$ 设 $PH = 4k, DH = 3k, k > 0$, 由于点 P 在 $x = 4$ 的右侧, $\therefore P$ 点坐标为 $(4 + 4k, 5 - 3k)$.

$$\because \text{点 } P \text{ 在抛物线上}, \therefore 5-3k = -\frac{1}{8}(4+4k)^2 + (4+4k) + 3. \quad (1 \text{ 分})$$

$$\text{整理得 } 2k^2 - 3k = 0, \therefore k = \frac{3}{2}, \text{ 或 } k = 0(\text{舍}). \quad (1 \text{ 分})$$

$$\therefore P\left(10, \frac{1}{2}\right). \quad (1 \text{ 分})$$

$$\text{方法二: 设直线 } AB \text{ 的解析式: } y = kx + b, \text{ 解得 } y = -\frac{3}{4}x + 3 \quad (1 \text{ 分})$$

$$\because DP \parallel AB, \therefore \text{设直线 } DP \text{ 的解析式为 } y = mx + n, \text{ 解得 } y = -\frac{3}{4}x + 8. \quad (1 \text{ 分})$$

$$\therefore \begin{cases} y = -\frac{1}{8}x^2 + 8x + 3, \\ y = -\frac{3}{4}x + 8. \end{cases} \quad \text{解得: } \begin{cases} x = 10, \\ y = \frac{1}{2}. \end{cases} \quad (1 \text{ 分})$$

$$\therefore P\left(10, \frac{1}{2}\right). \quad (1 \text{ 分})$$

(3) 情况一: 点 G 在 x 轴下方, 记为 G_1 ,

$$\because \text{点 } G_1 \text{ 在直线 } BD \text{ 上}, \therefore \angle ABD = \angle BAG_1 + \angle AG_1B,$$

$$\because \angle AG_1B = \frac{1}{2}\angle ABD, \therefore \angle BAG_1 = \angle AG_1B.$$

$$\therefore AB = BG_1, \because AB = 5, \therefore BG_1 = 5. \quad (1 \text{ 分})$$

$$\therefore S_{\triangle ABG_1} = \frac{1}{2} \times 5 \times 4 = 10. \quad (1 \text{ 分})$$

情况二: 点 G 在 x 轴上方, 记为 G_2 ,

作 $AF \perp BD$ 于点 F , 取 G_2 , 使 $G_2F = G_1F$, $\therefore AG_2 = AG_1$. $\therefore \angle AG_1B = \angle AG_2B$.

$$\because \angle AG_1B = \frac{1}{2}\angle ABD, \therefore \angle AG_2B = \frac{1}{2}\angle ABD. \therefore \text{点 } G_2 \text{ 符合题意.}$$

$$\because BG_1 = 5, A(0, 3), \therefore FG_1 = 8. \therefore FG_2 = 8. \therefore FB = 11. \quad (1 \text{ 分})$$

$$\therefore S_{\triangle ABG_2} = \frac{1}{2} \times 11 \times 4 = 22. \quad (1 \text{ 分})$$

25. 解: (1) 过 C 作 $CE \perp AB$ 于点 E , 联结 CO , 则 $CO = BO = 3$.

$$\because \tan B = 2\sqrt{2}, \therefore \text{设 } BE = a, CE = 2\sqrt{2}a. \therefore CB = 3a. \quad (1 \text{ 分})$$

$$\therefore \text{在 } \triangle COE \text{ 中}, OC^2 = OE^2 + EC^2 \text{ 即 } 9 = (3-a)^2 + 8a^2. \quad (1 \text{ 分})$$

$$\therefore a = \frac{2}{3}. \quad (1 \text{ 分})$$

$$\therefore CB = 3a = 2. \quad (1 \text{ 分})$$

$$(2) \text{ 方法一: } \because \triangle MBC \text{ 与 } \triangle MOC \text{ 相似}, \angle M = \angle M, \therefore \frac{MC}{MB} = \frac{MO}{MC} = \frac{CO}{BC}.$$

$$\because BC = 2, OC = 3, \therefore \frac{MC}{MB} = \frac{2}{3}. \quad (1 \text{ 分})$$

$$\therefore \text{设 } MC = 2x, MB = 3x, \therefore (3x)^2 = 2x(2x+3), \text{ 解得 } x = \frac{6}{5}, \therefore MC = \frac{18}{5}, MB = \frac{12}{5}. \quad (2 \text{ 分})$$

$$\text{作 } OH \perp DC \text{ 于点 } H, \because O \text{ 为圆心}, CD \text{ 是弦}, \therefore CH = DH. \quad (1 \text{ 分})$$

设 $CH=k$, 则在 $Rt\triangle MHO$ 中, $OM^2 = MH^2 + OH^2$.

$$\text{即} \left(3 + \frac{12}{5}\right)^2 = \left(\frac{18}{5} + k\right)^2 + 9 - k^2, \text{解得 } k=1, \therefore CD=2CH=2. \quad (1 \text{ 分})$$

方法二: 联结 CO , $\because DO=CO$, $\therefore \angle D = \angle OCD$, 同理 $\angle OCB = \angle OBC$,

设 $\angle D = \angle OCD = \alpha$, $\angle OCB = \angle OBC = \beta$,

$$\therefore \angle DOB = 180^\circ - 2\alpha + 180^\circ - 2\beta = 360^\circ - 2(\alpha + \beta).$$

$$\therefore \angle BCM = 180^\circ - (\alpha + \beta), \therefore \angle BCM = \frac{1}{2} \angle DOB. \quad (2 \text{ 分})$$

$$\because \triangle MBC \text{ 与 } \triangle MOC \text{ 相似}, \angle M = \angle M, \therefore \angle BCM = \angle COB. \quad (1 \text{ 分})$$

$$\therefore \angle COB = \frac{1}{2} \angle DOB. \because \angle DOC = \frac{1}{2} \angle DOB \quad \therefore \angle DOC = \angle COB. \quad (1 \text{ 分})$$

$$\therefore DC = BC. \because CB = 2, \therefore DC = 2. \quad (1 \text{ 分})$$

(3) 方法一: 延长 ON 交 BC 的延长线于点 G , 过点 G 作 $GQ \perp AB$ 于点 Q ,

$\because ON$ 平分 $\angle DOB$, $\therefore \angle GOB = \angle GOD$.

$\because OD \parallel BC$, $\therefore \angle G = \angle GOD$. $\therefore \angle GOB = \angle G$. $\therefore GB = OB = 3$.

$$\because CB = 2, \therefore GC = 1. \quad (1 \text{ 分})$$

在 $\triangle GOB$ 中, $\because \tan B = 2\sqrt{2}$, $GB = 3$, $\therefore GQ = 2\sqrt{2}$, $BQ = 1$.

$$\therefore OQ = 2, \therefore OG = 2\sqrt{3}. \quad (2 \text{ 分})$$

$$\because BC \parallel OD, \therefore \frac{GC}{DO} = \frac{GN}{NO}, \text{即 } \frac{GC}{DO} = \frac{OG - ON}{NO}. \quad (1 \text{ 分})$$

$$\therefore \frac{1}{3} = \frac{2\sqrt{3} - ON}{NO}, \therefore ON = \frac{3\sqrt{3}}{2}. \quad (1 \text{ 分})$$

方法二: 联结 OC , 作 $DQ \perp OC$,

$\because OC = OB$, $\therefore$ 设 $\angle B = \angle OCB = \beta$, $\because OD = OC$, $\therefore$ 设 $\angle D = \angle OCD = \alpha$.

$$\because OD \parallel BC, \therefore \alpha + \beta + \alpha = 180^\circ. \therefore \alpha = 90^\circ - \frac{\beta}{2}.$$

$$\because OD \parallel BC, \therefore \angle DOA = \angle B = \beta. \therefore \angle DOB = 180^\circ - \beta.$$

$$\because ON \text{ 平分 } \angle DOB, \therefore \angle DON = 90^\circ - \frac{\beta}{2}. \therefore \angle DON = \angle D. \therefore ND = NO. \quad (1 \text{ 分})$$

$\because OD \parallel BC, \therefore \angle DOC = \angle OCB = \angle B$.

$$\because \tan \angle ABC = 2\sqrt{2}, OD = 3, \therefore OQ = 1, DQ = 2\sqrt{2}.$$

$$\therefore DC = 2\sqrt{3}. \quad (2 \text{ 分})$$

在 $\triangle DOC$ 中, $\because \angle DON = \angle D$, $\angle OCD = \angle D$, $\therefore \angle DON = \angle OCD$.

$$\because \angle D = \angle D, \therefore \triangle DOC \sim \triangle DNO. \therefore \frac{DO}{DN} = \frac{DC}{DO}. \quad (1 \text{ 分})$$

$$\therefore \frac{3}{DN} = \frac{2\sqrt{3}}{3}, \therefore DN = \frac{3\sqrt{3}}{2}, \therefore ON = \frac{3\sqrt{3}}{2} \quad (1 \text{ 分})$$

黄浦区中考数学质量抽查试卷·参考答案

一、选择题(本大题 6 小题,每小题 4 分,满分 24 分)

1. B 2. D 3. C 4. B 5. D 6. A

二、填空题:(本大题共 12 题,每题 4 分,满分 48 分)

7. 2 8. 1 9. $4a^2 - b^2$ 10. $x=2$ 11. $\frac{4}{9}$ 12. 3 13. 35 14. -1 15. 6 16. $\frac{1}{4}\bar{b} - \frac{1}{4}\bar{a}$

17. 12 18. 4:3

三、解答题:(本大题共 7 题,满分 78 分)

$$19. \text{解:原式} = \frac{1}{x-2} \times \frac{(x-2)(x+2)}{x} - \frac{1+x}{x(x+1)} \quad (2 \text{分})$$

$$= \frac{x+2}{x} - \frac{1}{x} \quad (2 \text{分})$$

$$= \frac{x+1}{x}. \quad (2 \text{分})$$

把 $x=\sqrt{2}-1$ 代入上式,

$$\text{原式} = \frac{\sqrt{2}}{\sqrt{2}-1} \quad (2 \text{分})$$

$$= 2 + \sqrt{2}. \quad (2 \text{分})$$

$$20. \text{解:由②得,}(x-5y)(x+y)=0, \quad (2 \text{分})$$

$\therefore x-5y=0$ 或 $x+y=0$, 所以,原方程组可化为

$$\begin{cases} x^2 + y^2 = 26, \\ x-5y=0; \end{cases} \begin{cases} x^2 + y^2 = 26, \\ x+y=0. \end{cases} \quad (4 \text{分})$$

$$\text{解,得} \begin{cases} x_1=5, \\ y_1=1; \end{cases} \begin{cases} x_2=-5, \\ y_2=-1; \end{cases} \begin{cases} x_3=\sqrt{13}, \\ y_3=-\sqrt{13}; \end{cases} \begin{cases} x_4=-\sqrt{13}, \\ y_4=\sqrt{13}. \end{cases}$$

$$\text{所以原方程组的解是} \begin{cases} x_1=5, \\ y_1=1; \end{cases} \begin{cases} x_2=-5, \\ y_2=-1; \end{cases} \begin{cases} x_3=\sqrt{13}, \\ y_3=-\sqrt{13}; \end{cases} \begin{cases} x_4=-\sqrt{13}, \\ y_4=\sqrt{13}. \end{cases} \quad (4 \text{分})$$

$$21. \text{解:(1) 设一次函数解析式为 } y=2x+b, \quad (2 \text{分})$$

$$\because \text{该一次函数的图像经过点 } P(3,5), \therefore 2 \times 3 + b = 5, \quad (2 \text{分})$$

$$\therefore b = -1, \quad (1 \text{分})$$

$$\therefore y = 2x - 1. \quad (1 \text{分})$$

(2) $\because$ 点 $Q(x,y)$ 在该直线上,且在 x 轴的下方,

$$\therefore 2x - 1 < 0, \quad (2 \text{分})$$

$$x < \frac{1}{2}. \quad (1 \text{分})$$

$$\text{所以, } x \text{ 的取值范围是 } x < \frac{1}{2}. \quad (1 \text{分})$$

$$22. \text{解:过点 } O \text{ 作 } OE \perp CD, \text{垂足为点 } E. \quad (1 \text{分})$$

$$\therefore CE = DE. \quad (2 \text{ 分})$$

$$\text{在 Rt}\triangle PEO \text{ 中, } \because OP = 10, \sin \angle BPC = \frac{OE}{OP} = \frac{3}{5}, \therefore OE = 6, \quad (2 \text{ 分})$$

$$\because AB = 16, \therefore OC = \frac{1}{2} AB = 8, \quad (1 \text{ 分})$$

$$\text{在 Rt}\triangle COE \text{ 中, } OE^2 + CE^2 = CO^2, \quad (1 \text{ 分})$$

$$\therefore CE = \sqrt{64 - 36} = 2\sqrt{7}, \quad (2 \text{ 分})$$

$$\therefore CD = 2CE = 4\sqrt{7}. \quad (1 \text{ 分})$$

$$23. \text{ 解: (1) } \because CD = CE, \angle 1 = \angle 2, \angle C = \angle C, \therefore \triangle CDB \cong \triangle CEA, \quad (1 \text{ 分})$$

$$\therefore AC = BC, AE = BD, \quad (2 \text{ 分})$$

$$\therefore \frac{CD}{AC} = \frac{CE}{BC}, \therefore AB \parallel DE, \quad (2 \text{ 分})$$

$$\text{又 } \because AD \text{ 与 } BE \text{ 不平行, } \therefore \text{ 四边形 } ABED \text{ 是等腰梯形.} \quad (1 \text{ 分})$$

$$(2) \because AC = BC, \therefore \angle CAB = \angle CBA, \because \angle 1 = \angle 2, \therefore \angle OBA = \angle OAB, \quad (1 \text{ 分})$$

$$\therefore \angle AOD = 2\angle OBA, \text{ 又 } \angle AOD = 2\angle 1, \therefore \angle OBA = \angle 1, \quad (1 \text{ 分})$$

$$\because AB \parallel DE, \therefore \angle OBA = \angle BDE, \therefore \angle 1 = \angle BDE, \therefore DE = BE, \quad (1 \text{ 分})$$

$$\because AB \parallel DE, \therefore \frac{DE}{AB} = \frac{CE}{BC}, \quad (2 \text{ 分})$$

$$\text{又 } EC = 2, BE = 1, \therefore \frac{1}{AB} = \frac{2}{3}, \therefore AB = \frac{3}{2}. \quad (1 \text{ 分})$$

24. 解: (1) 由题意知

$$\begin{cases} a + b + c = 0, \\ 16a + 4b + c = 0, \\ c = 2. \end{cases} \text{ 解, 得 } \begin{cases} a = \frac{1}{2}, \\ b = -\frac{5}{2}, \\ c = 2. \end{cases} \quad (2 \text{ 分})$$

$$\therefore \text{ 抛物线的表达式为 } y = \frac{1}{2}x^2 - \frac{5}{2}x + 2. \quad (1 \text{ 分})$$

$$(2) \because OB = 4, OA = 1, OC = 2, \therefore \frac{OC}{OA} = \frac{OB}{CO} = 2, \quad (1 \text{ 分})$$

$$\because \angle COA = \angle BOC = 90^\circ, \quad (1 \text{ 分})$$

$$\therefore \triangle COA \sim \triangle BOC, \therefore \angle CAO = \angle BCO. \quad (1 \text{ 分})$$

$$(3) \because \angle PCB + \angle ACB = \angle BCO, \text{ 又 } \angle OCA + \angle ACB = \angle BCO, \therefore \angle PCB = \angle OCA, \quad (1 \text{ 分})$$

$$\because \triangle COA \sim \triangle BOC, \therefore \angle CBO = \angle OCA, \therefore \angle PCB = \angle CBO, \quad (1 \text{ 分})$$

① 若点 P 在 x 轴上方,

$$\because \angle PCB = \angle CBO, \therefore CP \parallel x \text{ 轴}, \quad (1 \text{ 分})$$

$$\therefore \text{ 直线 } CP \text{ 的表达式是 } y = 2; \quad (1 \text{ 分})$$

② 若点 P 在 x 轴下方,

设 CP 交 x 轴于点 $D(m, 0)$

$$\because \angle PCB = \angle CBO, \therefore CD = BD, \quad (1 \text{ 分})$$

$$\therefore m^2 + 2^2 = (4-m)^2, m = \frac{3}{2}, \therefore D\left(\frac{3}{2}, 0\right). \quad (1 \text{ 分})$$

$$\therefore \text{直线 } CP \text{ 的表达式为 } y = -\frac{4}{3}x + 2. \quad (1 \text{ 分})$$

综上所述, 直线 CP 的表达式为 $y = 2$ 或 $y = -\frac{4}{3}x + 2$.

$$25. \text{ 解}(1) \because AE \perp BD, BE = DE, \therefore AB = AD, \quad (1 \text{ 分})$$

$$\because \angle ACB = 90^\circ, AC = 1, BC = 7, \therefore AB = 5\sqrt{2}, \therefore AD = 5\sqrt{2},$$

$$\because \angle ACB = 90^\circ, \therefore \angle D + \angle CBD = 90^\circ, \because AE \perp BD, \therefore \angle EAD + \angle D = 90^\circ,$$

$$\therefore \angle CBD = \angle EAD, \quad (1 \text{ 分})$$

$$\because BF \parallel CD, \therefore \angle F = \angle EAD, \therefore \angle F = \angle CBD, \quad (1 \text{ 分})$$

$$\therefore \tan \angle AFB = \tan \angle CBD = \frac{CD}{BC} = \frac{1+5\sqrt{2}}{7}. \quad (1 \text{ 分})$$

$$(2) CE \cdot AF \text{ 的值不变}. \quad (1 \text{ 分})$$

$$\because \angle AED = \angle BCD = 90^\circ, \therefore \cos \angle D = \frac{ED}{AD} = \frac{CD}{BD}, \quad (1 \text{ 分})$$

$$\text{又 } \angle D = \angle D, \therefore \triangle CDE \sim \triangle BDA, \therefore \angle ECD = \angle ABD, \quad (1 \text{ 分})$$

$$\because \angle ECD + \angle BCE = 90^\circ, \angle ABD + \angle BAE = 90^\circ, \therefore \angle BCE = \angle BAE, \quad (1 \text{ 分})$$

$$\because \angle F = \angle CBD, \therefore \triangle CBE \sim \triangle AFB, \therefore \frac{CE}{AB} = \frac{CB}{AF}, \quad (1 \text{ 分})$$

$$\therefore CE \cdot AF = AB \cdot BC = 5\sqrt{2} \times 7 = 35\sqrt{2}. \quad (1 \text{ 分})$$

$$(3) \because \triangle BGE \text{ 与 } \triangle BAF \text{ 相似, 又 } \triangle CBE \sim \triangle BAF, \therefore \triangle BGE \sim \triangle CBE, \therefore \angle BEG = \angle BEC,$$

$$\angle GBE < \angle CBE, \therefore \angle GBE = \angle BCE, \text{ 又 } \angle GBE = \angle ECD, \therefore \angle BCE = \angle ECD,$$

$$\because \angle BCD = 90^\circ, \therefore \angle BCE = \angle ECD = 45^\circ, \quad (1 \text{ 分})$$

$$\therefore \angle BAE = 45^\circ, \because \angle BEA = 90^\circ, AB = 5\sqrt{2}, \therefore BE = AE = 5, \quad (1 \text{ 分})$$

过点 B 作 $BH \perp CE$ 于点 H .

$$\therefore BH = CH = \frac{7\sqrt{2}}{2}, HE = \frac{\sqrt{2}}{2}, \therefore CE = 4\sqrt{2}, \quad (1 \text{ 分})$$

$$\therefore CE \cdot AF = 35\sqrt{2}, \therefore AF = \frac{35}{4}. \quad (1 \text{ 分})$$

松江区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共 6 题,每题 4 分,满分 24 分)

1. B 2. D 3. C 4. C 5. A 6. D

二、填空题:(本大题共 12 题,每题 4 分,满分 48 分)

7. $a(2a-3)$ 8. $x \neq 1$ 9. $2\vec{a} + \vec{b}$ 10. $m \leq 1$ 11. $x > 2$ 12. $y = (x+3)^2$ 13. $y_1 < y_2$

14. $y^2 + y - 3 = 0$ 15. 19.6 16. $\frac{3}{10}$ 17. $289(1-x)^2 = 256$ 18. 2.5

三、解答题:(本大题共 7 题,满分 78 分)

19. 解:原式 $= 9 - (\sqrt{2} - 1) + 1 + \sqrt{2}$ (每个 2 分)
 $= 11$ 2 分

20. 解方程组: $\begin{cases} x + 2y = 12, & \text{①} \\ x^2 - 3xy + 2y^2 = 0. & \text{②} \end{cases}$

解:由②得: $(x - y)(x - 2y) = 0$.

$\therefore x - y = 0$ 或 $x - 2y = 0$. 2 分

原方程组可化为 $\begin{cases} x + 2y = 12, \\ x - y = 0, \end{cases}$ $\begin{cases} x + 2y = 12, \\ x - 2y = 0. \end{cases}$ 4 分

解这两个方程组,得原方程组的解为 $\begin{cases} x_1 = 4, \\ y_1 = 4, \end{cases}$ $\begin{cases} x_2 = 6, \\ y_2 = 3. \end{cases}$ 4 分

另解:由①得 $x = 12 - 2y$. ③ 1 分

把③代入②,得 $(12 - 2y)^2 - 3(12 - 2y)y + 2y^2 = 0$. 1 分

整理,得 $y^2 - 7y + 12 = 0$. 2 分

解得 $y_1 = 4, y_2 = 3$. 2 分

分别代入③,得 $x_1 = 4, x_2 = 6$. 2 分

$\therefore$ 原方程组的解为 $\begin{cases} x_1 = 4, \\ y_1 = 4, \end{cases}$ $\begin{cases} x_2 = 6, \\ y_2 = 3. \end{cases}$ 2 分

21. 解:(1) 设 $y = kx + b (k \neq 0)$,依题意,得

$x = -40$ 时, $y = -40$; $x = 0$ 时, $y = 32$ 2 分

代入,得 $\begin{cases} -40k + b = -40 \\ b = 32 \end{cases}$ 2 分

解得 $\begin{cases} k = \frac{9}{5} \\ b = 32 \end{cases}$ 2 分

$\therefore y = \frac{9}{5}x + 32$ 1 分

(2) 由 $y = 104$ 得, $\frac{9}{5}x + 32 = 104$, 2 分;

$\frac{9}{5}x = 72, x = 40$ 1 分

答:温度表上摄氏温度为 40 度.

22. 解:(1) 过点 O 作 $OH \perp AG$ 于点 H, 联接 OF. 1 分

$AB = AC = 10, AD \perp BC, BC = 12$

$\therefore BD = CD = \frac{1}{2}BC = 6$, 1 分

$\therefore AD = 8, \cos \angle BAD = \frac{4}{5}$

$\therefore AG = AD, OH \perp AG$

$$\therefore S_{\triangle APC} = S_{\text{四边形}AOCF} - S_{\triangle AOC} = S_{\text{梯形}OCPG} + S_{\triangle APG} - S_{\triangle AOC} = 14 + 13.5 - 12.5 = 15 \quad 3 \text{ 分}$$

$$(3) \angle CAB = \angle OAQ, AB = 6, AO = 6, AC = 5\sqrt{2},$$

$$\textcircled{1} \triangle ABC \sim \triangle AOQ \quad \therefore \frac{AB}{AC} = \frac{AO}{AQ} \quad \therefore AQ = \frac{25\sqrt{2}}{6} \quad 1 \text{ 分}$$

$$Q_1\left(\frac{5}{6}, \frac{25}{6}\right) \quad 1 \text{ 分}$$

$$\textcircled{2} \triangle ABC \sim \triangle AQO \quad \therefore \frac{AB}{AC} = \frac{AQ}{AO} \quad \therefore AQ = 3\sqrt{2} \quad 1 \text{ 分}$$

$$Q_2(2, 3) \quad 1 \text{ 分}$$

$$\therefore \text{点 } Q \text{ 的坐标 } Q_1\left(\frac{5}{6}, \frac{25}{6}\right) Q_2(2, 3) \text{ 时, } \triangle ABC \text{ 与 } \triangle AOQ \text{ 相似.}$$

25. 解: (1) 作 $AG \perp BC$ 于点 G ,

$$\therefore \angle BGA = 90^\circ$$

$$\because \angle BCD = 90^\circ, AD \parallel BC,$$

$$\therefore AG = DC = 6,$$

$$\because \tan \angle ABC = \frac{AG}{BG} = 2$$

$$\therefore BG = 3,$$

$$\because BC = 11$$

$$\therefore GC = 8,$$

$$\therefore AD = GC = 8$$

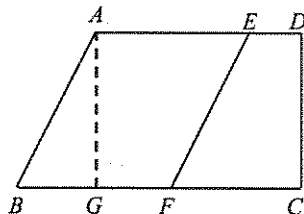
$$\therefore AE = 3ED$$

$$\therefore AE = 6, ED = 2$$

$$\because AD \parallel BC, AB \parallel EF$$

$$\therefore BF = AE = 6$$

$$\therefore CF = BC - BF = 5$$



(1 分)

(2) 过点 M 作 $PQ \perp CD$, 分别交 AB, CD, AG 于点 P, Q, H , 作 $MR \perp BC$ 于点 R

$$\text{易得 } GH = CQ = MR$$

$$\because MF \cos \angle EFC = x,$$

$$\therefore FR = x$$

$$\because \tan \angle ABC = 2$$

$$\therefore GH = MR = CQ = 2x$$

$$\because BG = 3, \text{ 由 } BF = 6 \text{ 得 } GF = 3$$

$$\therefore HM = 3 + x, MQ = CF - FR = 5 - x, AH = AG - GH = 6 - 2x$$

$$\because \angle AMQ = \angle AHM + \angle MAH, \text{ 且 } \angle AMN = \angle AHM = 90^\circ$$

$$\therefore \angle MAH = \angle NMQ$$

$$\therefore \triangle AHM \sim \triangle MQN$$

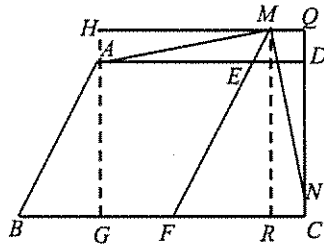
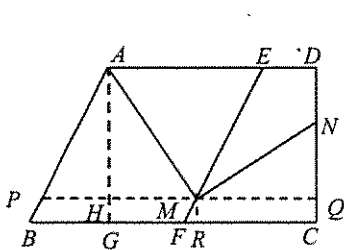
$$\therefore \frac{AH}{MQ} = \frac{HM}{NQ}, \text{ 即 } \frac{6-2x}{5-x} = \frac{3+x}{y-2x}$$

$$\therefore y = \frac{5x^2 - 14x - 15}{2x - 6}$$

$$\text{定义域: } 0 \leq x \leq 1$$

(3) ① $\angle AMN = 90^\circ$

1) 当点 M 在线段 EF 上时,



$\because \triangle AHM \sim \triangle MQN$ 且 $AM = MN$,

$\therefore AH = MQ$

(1分)

$$\therefore 6 - 2x = 5 - x,$$

$$\therefore x = 1$$

$$\therefore FM = \sqrt{5}$$

(1分)

2) 当点 M 在 FE 的延长线上时

同上可得 $AH = MQ$

$$\therefore 2x - 6 = 5 - x$$

$$\therefore x = \frac{11}{3}$$

$$\therefore FM = \frac{11}{3}\sqrt{5}$$

(2分)

② $\angle ANM = 90^\circ$

过点 N 作 $PQ \perp CD$, 分别交 AB 、 AG 于点 P 、 H , 作 $MR \perp BC$ 于交 BC 延长线于交直线 PN 于点 Q ,

$\because AN = MN$, 易得 $\triangle AHN \cong \triangle NQM$

$\therefore AH = NQ, HN = MQ = 8$

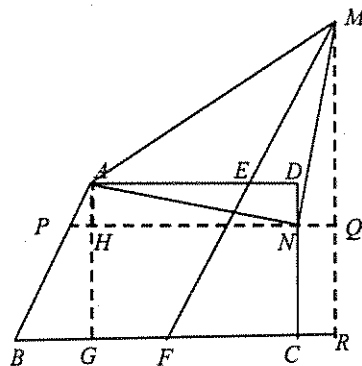
令 $PH = a$, 则 $AH = 2a, DN = 2a, CN = 6 - 2a$

$$\therefore FR = 5 + 2a, MR = 8 + (6 - 2a) = 14 - 2a$$

由 $MR = 2FR$ 得 $a = \frac{2}{3}$,

$$\therefore FR = \frac{19}{3}, MR = \frac{38}{3} \therefore FM = \frac{19}{3}\sqrt{5}$$

(1分)



徐汇区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共 6 题,每题 4 分,满分 24 分)

1. B 2. C 3. C 4. D 5. B 6. A

二、填空题:(本大题共 12 题,满分 48 分)

7. $2a^2b$ 8. $2m^2 - 6m$ 9. $x = 5$ 10. 1 11. $\frac{1}{3}b - \frac{2}{3}a$ 12. $\frac{400}{x-10} - \frac{400}{x} = 2$ 13. 0.21

14. 答案不唯一,如: $AC=BD$ 等 15. 4 16. $x > -1$ 17. 2200 18. $\frac{16}{5}$

三、(本大题共 7 题,第 19、20、21、22 题每题 10 分,第 23、24 题每题 12 分,第 25 题 14 分,满分 78 分)

19. 解:原式 $= \pi - 3 + 1 - |\sqrt{3} - 1| + \sqrt{3} - 1$; (5 分)

$$= \pi - 3 - \sqrt{3} + 1 + \sqrt{3}; \quad (3 \text{ 分})$$

$$= \pi - 2. \quad (2 \text{ 分})$$

20. 解:由方程②得 $2x - y = \pm 2$; (2 分)

与方程①组合得方程组;

$$(I) \begin{cases} x - y = 1, \\ 2x - y = 2 \end{cases} \text{ 或 } (II) \begin{cases} x - y = 1, \\ 2x - y = -2; \end{cases} \quad (4 \text{ 分})$$

$$\text{解方程组 (I)、(II) 得 } \begin{cases} x = 1, \\ y = 0 \end{cases} \text{ 或 } \begin{cases} x = -3, \\ y = -4; \end{cases} \quad (4 \text{ 分})$$

$$\therefore \text{原方程组的解是 } \begin{cases} x_1 = 1, \\ y_1 = 0 \end{cases} \text{ 或 } \begin{cases} x_2 = -3, \\ y_2 = -4. \end{cases}$$

21. 解:(1) 由题意,得 $\frac{1}{2} \times 1^2 + b + 2 = 0$; (1 分)

$$\text{解得 } b = -\frac{5}{2}; \quad (1 \text{ 分})$$

$$\therefore \text{抛物线的表达式是 } y = \frac{1}{2}x^2 - \frac{5}{2}x + 2; \quad (1 \text{ 分})$$

$$\text{顶点 } D\left(\frac{5}{2}, -\frac{9}{8}\right). \quad (2 \text{ 分})$$

(2) 由题意,得 $B(4, 0)$ 和 $C(0, 2)$; (2 分)

$$\therefore S_{\triangle CDB} = S_{\triangle ABC} + S_{\triangle ADB} = \frac{1}{2} \times 3 \times 2 + \frac{1}{2} \times 3 \times \frac{9}{8} = \frac{75}{16}. \quad (3 \text{ 分})$$

22. 解:(1) $OA = \frac{\sqrt{3}}{2}a$; (2 分)

$$(2) h_n = na, h'_n = \frac{\sqrt{3}}{2}(n-1)a + a; \quad (\text{各 } 2 \text{ 分})$$

(3) 按方案二在该种集装箱中装运铜管数多. (1 分)

由题意,按方案一装运铜管数 $= 25 \times 25 = 625$ (根); (1 分)

$$\therefore \frac{\sqrt{3}}{2}(n-1) \times 0.1 + 0.1 \leq 2.5, \text{ 即 } 0.0865n \leq 2.4865;$$

得 $n \leq 28.75$, 又 n 是整数, $\therefore n$ 的最大值是 28; (1 分)

$\therefore$ 按方案二装运铜管数 $= 14 \times 25 + 14 \times 24 = 686$ (根). (1 分)

23. 证明:(1) $\because AB=AC, \therefore \angle ACB = \angle ABC$; (1 分)

$$\because BD=ED, \therefore \angle BED = \angle DBE; \quad (1 \text{ 分})$$

$$\therefore \angle ABC = \angle DBE, \therefore \angle ACB = \angle DEB, \therefore \triangle ABC \sim \triangle DBE; \quad (1 \text{ 分})$$

$$\therefore \frac{AB}{DB} = \frac{CB}{BE}; \quad (1 \text{ 分})$$

$$\text{又 } \angle ABC - \angle DBC = \angle DBE - \angle DBC, \text{ 即 } \angle ABD = \angle CBE;$$

$$\therefore \triangle ABD \sim \triangle CBE; \therefore \frac{CE}{BE} = \frac{AD}{BD} = 1; \quad (1 \text{ 分})$$

$$\therefore CE = BE. (1 \text{ 分})$$

$$(2) \because \angle ACB = \angle ABC = 72^\circ, \therefore \angle A = 180^\circ - 2 \times 72^\circ = 36^\circ; (1 \text{ 分})$$

$$\because AD = BD, \therefore \angle DBA = \angle A = 36^\circ; (1 \text{ 分})$$

$$\therefore \angle DBC = 72^\circ - 36^\circ = 36^\circ;$$

$$\because \triangle ABC \sim \triangle DBE, \therefore \angle EDB = \angle A = 36^\circ;$$

$$\therefore \angle EDB = \angle DBA, \therefore DE \parallel AB; (1 \text{ 分})$$

$$\because \triangle ABD \sim \triangle CBE, \therefore \angle ECB = \angle A = 36^\circ;$$

$$\therefore \angle ECB = \angle DBC, \therefore CE \parallel DB; (1 \text{ 分})$$

$$\therefore \text{四边形 } DBFE \text{ 是平行四边形}; (1 \text{ 分})$$

$$\text{又 } BD = DE, \therefore \text{四边形 } DBFE \text{ 是菱形}. (1 \text{ 分})$$

$$24. \text{解: (1) 过点 } A \text{ 作 } AG \perp OC, \text{垂足是 } G. \text{易得 } AG \parallel OD, \therefore \frac{AG}{OD} = \frac{CG}{OC} = \frac{AC}{CD} = \frac{1}{2};$$

$$\text{由题意, 得 } C(0, 4), \therefore OC = 4;$$

$$\text{在 } Rt\triangle DOC \text{ 中, } \angle DOC = 90^\circ, \tan \angle CDO = 2, \therefore OD = 2;$$

$$\therefore AG = 1, CG = 2; \therefore A(1, 6); (3 \text{ 分})$$

$$\therefore 6 = \frac{k}{1}, \text{得 } k = 6; \therefore y = \frac{6}{x}. (1 \text{ 分})$$

$$(2) \text{过点 } O \text{ 作 } OF \perp AB, \text{垂足是 } F.$$

$$\text{由题意, 得 } D(-2, 0); \therefore \text{直线 } AB \text{ 的表达式是 } y = 2x + 4; (1 \text{ 分})$$

$$\text{又点 } B \text{ 是直线 } AB \text{ 与双曲线 } y = \frac{6}{x} \text{ 的交点, } \therefore B(-3, -2), DB = \sqrt{5};$$

$$\text{在 } Rt\triangle DOC \text{ 中, 可解得 } OF = \frac{4\sqrt{5}}{5}, DF = \frac{2\sqrt{5}}{5}; (1 \text{ 分})$$

$$\therefore BF = \frac{7\sqrt{5}}{5}; (1 \text{ 分})$$

$$\text{在 } Rt\triangle BFO \text{ 中, } \angle BFO = 90^\circ, \tan \angle DBO = \frac{OF}{BF} = \frac{4}{7}. (1 \text{ 分})$$

$$(3) \text{以 } AB \text{ 分别为对角线和边两种情况讨论.}$$

$$1^\circ \text{当 } AB \text{ 是对角线时, 由题意, 可知直线 } x = -1 \text{ 与双曲线 } y = \frac{6}{x} \text{ 的交点就是点 } N, \therefore N(-1, -6); (2 \text{ 分})$$

$$2^\circ \text{当 } AB \text{ 是边时, 将 } AB \text{ 向右平移 2 个单位, 点 } B \text{ 落在直线 } x = -1 \text{ 上, } \therefore N(3, 2); (1 \text{ 分})$$

$$\text{当 } AB \text{ 是边时, 将 } AB \text{ 向左平移 2 个单位, 点 } A \text{ 落在直线 } x = -1 \text{ 上, } \therefore N(-5, -\frac{6}{5}); (1 \text{ 分})$$

$$\text{综合 } 1^\circ, 2^\circ, N(-1, -6) \text{ 或 } N(3, 2) \text{ 或 } N(-5, -\frac{6}{5}).$$

$$25. \text{解: (1) 过点 } O \text{ 作 } OF \perp BE, \text{垂足为 } F.$$

$$\text{设 } OA = x, \text{则 } OP = x - 1, OD = x + a; \because OA^2 = OP \cdot OD,$$

$$\text{即 } x^2 = (x - 1)(x + a), \text{解得 } x = \frac{a}{a - 1}; (1 \text{ 分})$$

$$\therefore OA = \frac{a}{a - 1}, OP = \frac{1}{a - 1}, OD = \frac{a^2}{a - 1};$$

$$\text{当 } a = 2 \text{ 时, 可得 } OA = 2, OD = 4, \therefore BD = 2\sqrt{5};$$

易得 $\triangle BOF \sim \triangle DOB$, $\therefore \frac{BF}{OB} = \frac{OB}{OD}$, 又 $OB = OA = 2$

$$\therefore BF = \frac{2\sqrt{5}}{5}, \therefore BE = \frac{4\sqrt{5}}{5}. \quad (3 \text{ 分})$$

(2) 当点 C 与点 A 重合时, $\frac{CD}{PC} = \frac{AD}{PA} = a$. (1 分)

当点 C 与点 A 不重合时, 联结 OC , $\because OC = OA$, $\therefore OC^2 = OP \cdot OD$;

即 $\frac{OP}{OC} = \frac{OC}{OD}$, 又 $\angle COP = \angle DOC$, $\therefore \triangle OCP \sim \triangle ODC$,

$$\therefore \frac{CD}{PC} = \frac{OD}{OC} = a, \therefore CD = aPC; \text{ 又 } a > 1, \therefore CD > PC; \quad (1 \text{ 分})$$

$\because \odot P$ 和 $\odot C$ 相切, PC 是圆心距, $\therefore \odot P$ 和 $\odot C$ 相只能内切;

$$\therefore CD - PC = PC; \text{ 即 } aPC - PC = PC; \quad (1 \text{ 分})$$

$$\text{解得 } a = 2. \quad (1 \text{ 分})$$

(3) 联结 BP, OC . $\because \triangle OCP \sim \triangle ODC$, $\therefore \angle OCP = \angle D$;

$$\because OC = OB, \therefore \angle OBC = \angle OCB; \therefore \angle D + \angle OBC = 90^\circ,$$

$$\therefore \angle OCP + \angle OCB = 90^\circ, \text{ 即 } \angle BCP = 90^\circ. \quad (1 \text{ 分})$$

$$\because PC \cdot OA = BC \cdot OP, OA = OB, \therefore \frac{PC}{BC} = \frac{OP}{OB};$$

$$\text{又 } \angle BOP = \angle BCP = 90^\circ, \therefore \triangle BOP \sim \triangle BCP; \quad (1 \text{ 分})$$

$$\therefore \frac{OB}{CB} = \frac{BP}{CP} = 1; \therefore CB = OB, \therefore CB = OB = OC;$$

$$\therefore \triangle OBC \text{ 是等边三角形}, \therefore \angle OBC = 60^\circ; \quad (1 \text{ 分})$$

$$\text{在 Rt}\triangle BOD \text{ 中}, \angle BOD = 90^\circ, \tan \angle DOB = \frac{OD}{OB} = a,$$

$$\text{即 } a = \tan 60^\circ = \sqrt{3}, OA = \frac{a}{a-1} = \frac{3+\sqrt{3}}{2}. \quad (2 \text{ 分})$$

闵行区中考数学质量抽查试卷 · 参考答案

一、选择题: (本大题共 6 题, 每题 4 分, 满分 24 分)

1. D 2. B 3. B 4. C 5. D 6. A

二、填空题: (本大题共 12 题, 每题 4 分, 满分 48 分)

7. 4 8. $a(a-\sqrt{2})(a+\sqrt{2})$ 9. $x = \frac{1}{2}$ 10. $-\frac{3}{5} < x \leq 3$ 11. $m < -\frac{1}{4}$ 12. -2

13. 矩形, 等腰梯形, 正方形 (任一均可) 14. $\frac{1}{2}\vec{a} + 2\vec{b}$ 15. $\frac{1}{3}$ 16. 15 17. 3 18. $\frac{13}{5}$

三、解答题: (本大题共 7 题, 满分 78 分)

19. 解: 原式 $= \sqrt{3} - \sqrt{2} - \frac{1}{2} + 1 - \sqrt{3}$ (8 分)

$$= \frac{1}{2} - \sqrt{2}. \quad (2 \text{ 分})$$

20. 解: $(x-4)(x-2) + 2x = x+2$. (2 分)

$$x^2 - 6x + 8 + 2x = x + 2. \quad (2 \text{ 分})$$

$$x^2 - 5x + 6 = 0. \quad (2 \text{ 分})$$

$$x_1 = 3, x_2 = 2. \quad (2 \text{ 分})$$

经检验 $x = 3$ 是原方程的解, $x = 2$ 是增根, 舍去. (1 分)

所以原方程的解是 $x = 3$. (1 分)

21. 解: (1) 过点 C 作 $CE \perp AB$, 垂足为点 E .

$$\because CE \perp AB, \therefore \angle CEB = \angle CEA = 90^\circ.$$

$$\text{在 Rt}\triangle CBE \text{ 中}, \because \angle ABC = 30^\circ, BC = 8, \therefore CE = 4. \quad (1 \text{ 分})$$

$$\text{利用勾股定理, 得 } BE = \sqrt{BC^2 - CE^2} = \sqrt{8^2 - 4^2} = 4\sqrt{3}. \quad (1 \text{ 分})$$

$$\text{在 Rt}\triangle CEA \text{ 中}, \because CE = 4, \sin \angle A = \frac{\sqrt{5}}{5}, \therefore AC = \frac{CE}{\sin \angle A} = \frac{4}{\frac{\sqrt{5}}{5}} = 4\sqrt{5}.$$

$$\therefore AE = \sqrt{AC^2 - CE^2} = \sqrt{(4\sqrt{5})^2 - 4^2} = 8. \quad (1 \text{ 分})$$

$$\therefore AB = AE + EB = 8 + 4\sqrt{3}. \quad (1 \text{ 分})$$

$$\therefore S_{\triangle ABC} = \frac{1}{2} AB \cdot CE = \frac{1}{2} \times (8 + 4\sqrt{3}) \times 4 = 16 + 8\sqrt{3}. \quad (1 \text{ 分})$$

(2) 过点 D 作 $DF \perp AB$, 垂足为点 F .

$$\because CE \perp AB, DF \perp AB, \therefore \angle DFA = \angle CEA = 90^\circ, \therefore DF \parallel CE. \quad (1 \text{ 分})$$

$$\text{又} \because BD \text{ 是 } AC \text{ 边上的中线}, \therefore \frac{AD}{AC} = \frac{DF}{CE} = \frac{AF}{AE} = \frac{1}{2}. \quad (1 \text{ 分})$$

$$\text{又} \because CE = 4, AE = 8, BE = 4\sqrt{3}, \therefore DF = 2, AF = 4, EF = 4. \quad (1 \text{ 分})$$

$$\therefore BF = 4 + 4\sqrt{3}. \quad (1 \text{ 分})$$

$$\text{在 Rt}\triangle DFB \text{ 中}, \therefore \cot \angle ABD = \frac{BF}{DF} = \frac{4 + 4\sqrt{3}}{2} = 2 + 2\sqrt{3}. \quad (1 \text{ 分})$$

22. 解: (1) 在 $\text{Rt}\triangle BEA$ 中, $AE^2 + BE^2 = AB^2$.

$$\because i = 1: \frac{5}{12}, \therefore \text{设 } AE = 5k, BE = 12k. \quad (1 \text{ 分})$$

$$\text{又} \because AB = 26, \therefore (5k)^2 + (12k)^2 = 26^2, \quad (1 \text{ 分})$$

$$\text{解得 } k = 2. \quad (1 \text{ 分})$$

$$\therefore AE = 10, BE = 24. \quad (1 \text{ 分})$$

答: 改造前坡顶与地面的距离 BE 的长为 24 米. (1 分)

(2) 过点 F 作 $FH \perp AD$, 垂足为点 H .

$$\because BC \parallel AD, BE \perp AD, FH \perp AD,$$

$$\therefore FH = BE = 24. \quad (1 \text{ 分})$$

$$\text{在 Rt}\triangle FHA \text{ 中}, \therefore \cot \angle FAH = \frac{AH}{FH}.$$

$$\text{又} \because \angle FAH = 53^\circ, \therefore \cot \angle FAH = \cot 53^\circ = \frac{AH}{24} \approx 0.75. \quad (1 \text{ 分})$$

$$\therefore AH = 18. \quad (1 \text{ 分})$$

$$\therefore HE = AH - AE = 18 - 10 = 8.$$

$$\because FH \parallel BE, BC \parallel AD, \therefore BF = EH = 8. \quad (1 \text{ 分})$$

答: BF 至少是 8 米. (1 分)

23. 证明: (1) $\because$ 矩形 $ABCD$, $\therefore AE \parallel CF$. $\therefore \angle AEO = \angle CFO$. (1分)
 又 $\because$ 点 O 为对角线 AC 的中点, $\therefore AO = CO$. (1分)
 又 $\because \angle AOE = \angle COF$, $\therefore \triangle EOA \cong \triangle FOC$. (1分)
 $\therefore EO = FO$. (1分)
 $\therefore$ 四边形 $AFCE$ 是平行四边形. (1分)
 又 $\because EF \perp AC$, $\therefore$ 四边形 $AFCE$ 是菱形. (1分)
- (2) $\because EO = FO, OF = 2GO$, $\therefore EG = GO$. (1分)
 $\because$ 矩形 $ABCD, EF \perp AC$, $\therefore \angle EDC = \angle EOC = 90^\circ$.
 又 $\because \angle EGD = \angle CGO$, $\therefore \triangle EGD \sim \triangle CGO$. (2分)
 $\therefore \frac{EG}{DG} = \frac{GC}{GO}$. (1分)
 又 $\because EG = GO$, $\therefore \frac{GO}{DG} = \frac{GC}{GO}$. (1分)
 $\therefore GO^2 = DG \cdot GC$. (1分)
24. 解: (1) 抛物线 $y = ax^2 + 2x + c$ 经过点 $C(0, 3)$,
 $\therefore c = 3$. (1分)
 抛物线 $y = ax^2 + 2x + 3$ 经过点 $A(-1, 0)$,
 $\therefore a \times (-1)^2 - 2 + 3 = 0$. 解得 $a = -1$.
 $\therefore$ 所求抛物线的关系式为 $y = -x^2 + 2x + 3$. (1分)
 抛物线的对称轴是直线 $x = 1$. (1分)
 顶点坐标 $M(1, 4)$. (1分)
- (2) 直线 $y = kx + b$ 经过 C, M 两点, 点 $C(0, 3)$, 点 $M(1, 4)$,
 $\therefore \begin{cases} b = 3 \\ 4 = k + b \end{cases}$, 解得 $\begin{cases} k = 1 \\ b = 3 \end{cases}$, $\therefore$ 直线 CD 的解析式为 $y = x + 3$. (1分)
 $\therefore$ 点 D 的坐标为 $(-3, 0)$. $\therefore AD = 2$. (1分)
 $\because$ 点 C 关于直线 l 的对称点为 N ,
 $\therefore$ 点 N 的坐标为 $(2, 3)$. (1分)
 $\therefore CN = 2 = AD$.
 又 $\because CN \parallel AD$, $\therefore$ 四边形 $CDAN$ 是平行四边形. (1分)
- (3) 过点 P 作 $PH \perp CD$, 垂足为点 H .
 $\because$ 以点 P 为圆心的圆经过 A, B 两点, 并且与直线 CD 相切,
 $\therefore PH = AP$, 即: $PH^2 = AP^2$. (1分)
 设点 P 的坐标为 $(1, t)$, $\therefore PM = |4 - t|$, $AP^2 = 2^2 + t^2$.
 $\because$ 在 $Rt\triangle MED$ 中, 点 D 的坐标为 $(-3, 0)$, 点 M 的坐标为 $(1, 4)$,
 $\therefore DE = ME = 4$. $\therefore \angle DME = 45^\circ$. $\therefore PH = MH = \frac{\sqrt{2}}{2} MP = \frac{\sqrt{2}}{2} |4 - t|$.
 即得 $4 + t^2 = \frac{1}{2} (4 - t)^2$. (1分)
 $\therefore$ 解得 $t = -4 \pm 2\sqrt{6}$. (1分)
 $\therefore$ 点 P 的坐标为 $(1, -4 + 2\sqrt{6})$ 或 $(1, -4 - 2\sqrt{6})$. (1分)

25. 解:(1) 过点 H 作 $HG \parallel CD$, 交 AB 于点 G .

$$\because AB = AC, AH \perp BC, \therefore BH = CH. \quad (1 \text{ 分})$$

$$\text{又} \because HG \parallel CD, AB = 6, AD = 2, \therefore DG = BG = 2. \quad (1 \text{ 分})$$

$$\text{又} \because HG \parallel CD, \therefore AE = EH = 2. \quad (1 \text{ 分})$$

$$\therefore AH = 4. \quad (1 \text{ 分})$$

(2) 联结 AP , 设 $BP = t$.

$\because$ 以点 P 为圆心, BP 为半径的圆与 $\odot A$ 外切,

$$\therefore AP = t + 2. \quad (1 \text{ 分})$$

$\because$ 以点 P 为圆心, CP 为半径的圆与 $\odot A$ 内切,

$$\therefore AP = PC - 2. \quad (1 \text{ 分})$$

$$\therefore PC = t + 4, \therefore BC = 2t + 4, \therefore BH = \frac{1}{2}BC = t + 2.$$

$$\therefore HP = 2. \quad (1 \text{ 分})$$

$$\text{在 Rt}\triangle ABH \text{ 中}, AH^2 = AB^2 - BH^2,$$

$$\text{在 Rt}\triangle APH \text{ 中}, AH^2 = AP^2 - HP^2,$$

$$\text{可得 } 6^2 - (t + 2)^2 = (t + 2)^2 - 2^2. \quad (1 \text{ 分})$$

$$\text{解得: } t = 2\sqrt{5} - 2 (\text{负值舍去})$$

$$\therefore BC = 4\sqrt{5}. \quad (1 \text{ 分})$$

另解: 联结 AP , 设 $BP = a, BC = b$.

$\because$ 以点 P 为圆心, BP 为半径的圆与 $\odot A$ 外切,

$$\therefore AP = a + 2. \quad (1 \text{ 分})$$

$\because$ 以点 P 为圆心, CP 为半径的圆与 $\odot A$ 内切,

$$\therefore AP = PC - 2. \quad (1 \text{ 分})$$

$$\therefore a + 2 = b - a - 2, \text{ 即 } b = 2a + 4. \quad \textcircled{1}$$

$$\text{在 Rt}\triangle APH \text{ 中}, AH^2 = AP^2 - HP^2,$$

$$\text{在 Rt}\triangle BCH \text{ 中}, AH^2 = AC^2 - CH^2,$$

$$\text{可得 } (a + 2)^2 - \left(\frac{1}{2}b - a\right)^2 = 36 - \left(\frac{1}{2}b\right)^2, \quad (1 \text{ 分})$$

$$\text{即: } 4a + ab - 32 = 0. \quad \textcircled{2}$$

$$\text{把方程} \textcircled{1} \text{ 代入方程} \textcircled{2} \text{ 得 } a^2 + 4a - 16 = 0$$

$$\text{解得: } a = 2\sqrt{5} - 2 (\text{负值舍去})$$

$$\therefore BC = b = 4\sqrt{5}. \quad (1 \text{ 分})$$

(3) 过点 B 作 $BM \parallel DF$, 交 AH 的延长线于点 M .

$$\because BM \parallel DF, AB = 6, AD = 2, DF = x,$$

$$\therefore \frac{AD}{AB} = \frac{AF}{AM} = \frac{DF}{BM} = \frac{1}{3}, \text{ 即: } BM = 3x, AM = 6. \quad (1 \text{ 分})$$

$$\text{设 } HM = k.$$

$$\text{在 Rt}\triangle ABH \text{ 中}, BH^2 = AB^2 - AH^2,$$

$$\text{在 Rt}\triangle BHM \text{ 中}, BH^2 = BM^2 - MH^2,$$

$$\therefore 6^2 - (6-k)^2 = (3x)^2 - k^2, \text{ 即 } k = \frac{3}{4}x^2,$$

$$\therefore BH^2 = (3x)^2 - \left(\frac{3}{4}x\right)^2, AH = 6 - \frac{3}{4}x^2. \quad (1 \text{ 分})$$

$$\therefore BC = 2BH = \frac{3}{2}x \sqrt{16 - x^2}. \quad (1 \text{ 分})$$

$$\therefore y = \frac{1}{2}BC \cdot AH = \frac{1}{2} \times \frac{3}{2}x \sqrt{16 - x^2} \cdot \left(6 - \frac{3}{4}x^2\right) = \frac{72x - 9x^3}{16} \sqrt{16 - x^2}.$$

$$\therefore y \text{ 关于 } x \text{ 的函数解析式为: } y = \frac{72x - 9x^3}{16} \sqrt{16 - x^2}, \quad (1 \text{ 分})$$

$$\text{自变量 } x \text{ 的取值范围为 } 0 < x < 4. \quad (1 \text{ 分})$$

虹口区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共 6 题,满分 24 分)

1. D 2. B 3. C 4. B 5. A 6. D

二、填空题:(本大题共 12 题,满分 48 分)

7. 2 8. $x=3$ 9. $m < 1$ 10. 答案不唯一,如 $x^2 + y = 3$ 等 11. $x \neq \frac{1}{2}$ 12. $<$ 13. $\frac{4}{7}$ 14. 9

15. 7π 16. 内切 17. $\frac{2\sqrt{3}}{3}$ 18. 6 或 $\frac{25}{6}$

三、解答题:(本大题共 7 题,满分 78 分)

$$19. \text{ 解:原式} = \frac{x}{x-4} + \frac{4}{(x+4)(x-4)} \cdot \frac{x+4}{2} = \frac{x}{x-4} + \frac{2}{x-4} = \frac{x+2}{x-4}$$

$$\text{当 } x=8 \text{ 时,原式} = \frac{8+2}{8-4} = \frac{10}{4} = \frac{5}{2}$$

20. 解:(1) 设所求二次函数的解析式为: $y = ax^2 + bx + c (a \neq 0)$, 由题意得:

$$\begin{cases} c = -1, \\ a + b + c = 5, \\ a - b + c = -3. \end{cases} \text{ 解得: } \begin{cases} a = 2, \\ b = 4, \\ c = -1. \end{cases}$$

$$\therefore \text{所求二次函数的解析式为 } y = 2x^2 + 4x - 1$$

$$(2) y = 2x^2 + 4x - 1 = 2(x^2 + 2x) - 1 = 2(x^2 + 2x + 1) - 2 - 1 = 2(x+1)^2 - 3$$

21. 解:过点 C 作 $CH \perp AB$, 垂足为点 H.

$$\therefore \sin B = \frac{\sqrt{2}}{2}, BC = 2\sqrt{2}$$

$$\therefore CH = BC \cdot \sin B = 2 \quad \therefore BH = 2$$

$$\text{在 Rt}\triangle AHC \text{ 中, } \therefore \tan A = \frac{1}{2}, \therefore \frac{CH}{AH} = \frac{1}{2}$$

$$\therefore AH = 2CH = 4$$

$$\therefore AB = AH + HB = 4 + 2 = 6$$

$$\therefore CD \text{ 是边 } AB \text{ 上的中线, } \therefore BD = \frac{1}{2}AB = 3,$$

$$\therefore DH = BD - BH = 3 - 2 = 1,$$

∴在 Rt△DHC 中, $CD = \sqrt{CH^2 + DH^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$,

$$\therefore \cos \angle CDB = \frac{DH}{CD} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}.$$

22. 解: 设这个班级共有 x 名同学, 则该班级实际参加制作环保包装盒的学生有 $(x-10)$ 名, 根据题意, 得:

$$\frac{600}{x-10} - \frac{600}{x} = 5$$

解这个方程, 得: $x_1 = 40, x_2 = -30$.

经检验, $x_1 = 40, x_2 = -30$ 都是原方程的根,

但 $x_2 = -30$ 不合题意, 舍去.

答: 这个班级共有 40 名同学.

23. 证明: (1) ∵ $AF \parallel EC$, ∴ $\angle AFB = \angle CED$,

$$\because AB \parallel DC, \therefore \angle ABF = \angle CDE,$$

$$\because BE = DF, \therefore BE + EF = DF + EF, \text{即 } BF = DE,$$

$$\therefore \triangle ABF \cong \triangle CDE, \therefore AB = DC,$$

$$\text{又} \because AB \parallel DC$$

$$\therefore \text{四边形 } ABCD \text{ 是平行四边形}$$

- (2) ∵ 四边形 $ABCD$ 是平行四边形,

$$\therefore AB = DC, AD \parallel BC, \therefore \frac{AD}{BH} = \frac{DF}{BF},$$

$$\because AB \parallel DC, \therefore \frac{DG}{AB} = \frac{DF}{BF}, \therefore \frac{DG}{AB} = \frac{AD}{BH}.$$

$$\because AB = DC, \therefore \frac{DG}{DC} = \frac{AD}{BH},$$

$$\therefore AD \cdot DC = BH \cdot DG.$$

24. 解: (1) 由题意, 得: $AO = 3, \angle AOB = 90^\circ, \tan \angle BAO = 2$,

$$\therefore BO = AO \cdot \tan \angle BAO = 3 \times 2 = 6, \therefore B(0, 6).$$

设直线 AB 的表达式为 $y = kx + b (k \neq 0)$,

∵ 直线 AB 过点 $A(3, 0), B(0, 6)$,

$$\therefore \begin{cases} 3k + b = 0, \\ b = 6. \end{cases} \text{解得: } \begin{cases} k = -2, \\ b = 6. \end{cases}$$

∴ 直线 AB 的表达式为 $y = -2x + 6$.

- (2) 过点 D 作 $DH \perp x$ 轴, 垂足为点 H , 则 $DH \parallel BO$.

$$\therefore \frac{OH}{OA} = \frac{BD}{BA},$$

$$\because AD = 2DB, \therefore \frac{BD}{AB} = \frac{1}{3}, \therefore \frac{OH}{OA} = \frac{1}{3}.$$

∵ $OA = 3, \therefore OH = 1, \therefore$ 点 D 的横坐标是 1,

把 $x = 1$ 代入 $y = -2x + 6$ 得: $y = 4, \therefore D(1, 4)$

∴ 反比例函数 $y = \frac{k_1}{x} (k_1 \neq 0)$ 的图像经过点 D ,

$$\therefore 4 = \frac{k_1}{1}, \therefore k_1 = 4.$$

$$(3) k_2 \text{ 的值为 } \frac{27}{16} \text{ 或 } -\frac{9}{2}.$$

25. 解: (1) 过点 A 作 $AH \perp FE$, 垂足为点 H,

$$\because AH \text{ 过圆心 } A, \therefore EF = 2EH,$$

$$\because \angle ACB = 90^\circ, ED \perp BC,$$

$$\therefore ED \parallel AC, \therefore \frac{ED}{AC} = \frac{BD}{BC},$$

$$\text{又 } \because D \text{ 为边 } BC \text{ 中点}, AC = 2, \therefore ED = \frac{1}{2} AC = 1,$$

$$\because AH \perp FE, ED \perp BC, \therefore \angle AHD = 90^\circ, \angle HDC = 90^\circ,$$

$$\text{又 } \because \angle ACB = 90^\circ, \therefore \text{四边形 } ACDH \text{ 是矩形},$$

$$\therefore DH = AC = 2,$$

$$\therefore EH = DH - ED = 2 - 1 = 1, \therefore EF = 2.$$

(2) 过点 A 作 $AH \perp FE$, 垂足为点 H,

$$\text{由(1)知: } EF = 2EH, DH = AC = 2, ED \parallel AC, \therefore \frac{DE}{AC} = \frac{BD}{BC},$$

$$\therefore \frac{DC}{BC} = x, \therefore \frac{BD}{BC} = 1 - x,$$

$$\because AC = 2, \therefore \frac{DE}{2} = 1 - x, \therefore DE = 2 - 2x.$$

$$\therefore EH = 2 - (2 - 2x) = 2x, \therefore EF = 4x,$$

$$\therefore \text{所求的函数解析式是 } y = 4x.$$

(3) $\because DE \parallel AC, DE$ 过 $\triangle ABC$ 的重心,

$$\therefore \frac{BE}{EA} = 2, \frac{ED}{AC} = \frac{2}{3}, \frac{CD}{BC} = \frac{1}{3},$$

$$\text{又 } \because AC = 2, \therefore ED = \frac{4}{3},$$

$$\text{由(2)知: } FE = \frac{4}{3}.$$

$$\therefore FD = FE + ED = \frac{8}{3}, \therefore \frac{ED}{FD} = \frac{1}{2},$$

$$\text{又 } \because \frac{CD}{BC} = \frac{1}{3}, \therefore \frac{CD}{BD} = \frac{1}{2}, \therefore \frac{ED}{FD} = \frac{CD}{BD},$$

$$\text{又 } \because ED \perp BC, \therefore \angle CDE = \angle BDF = 90^\circ,$$

$$\therefore \triangle CDE \sim \triangle BDF, \therefore \frac{CE}{BF} = \frac{CD}{BD} = \frac{1}{2}$$

设 $\odot A$ 的半径为 r , 则 $AF = AE = r, BE = 2r, AB = 3r$.

$$\because \angle AFB = 90^\circ, \therefore BF = \sqrt{AB^2 - AF^2} = \sqrt{8r^2} = 2\sqrt{2}r,$$

$$\therefore \frac{BF}{AB} = \frac{2\sqrt{2}r}{3r} = \frac{2\sqrt{2}}{3}, \therefore \frac{CE}{AB} = \frac{CE}{BF} \cdot \frac{BF}{AB} = \frac{1}{2} \times \frac{2\sqrt{2}}{3} = \frac{\sqrt{2}}{3}.$$

闸北区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共 6 题,每题 4 分,满分 24 分)

1. C 2. A 3. C 4. B 5. B 6. D

二、填空题:(本大题共 12 题,每题 4 分,满分 48 分)

7. a^3 8. $3x(x-2)$ 9. $1 < x < 3$ 10. $x \leq 1$ 11. 1 12. 4 13. 134 14. $100(1+x)^2 = 125$

15. 135 16. $2\vec{m} + 2\vec{n}$ 17. (2,0) 18. $\frac{8\sqrt{10}}{5}$

三、解答题:(本大题共 7 题,满分 78 分)

19. (本题满分 10 分)

$$\text{解:原式} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}+1}{2} + \sqrt{3} - 1 - 3$$

2 分 $\times 4 = 8$ 分

$$= 2\sqrt{3} - \frac{7}{2}$$

2 分

20. (本题满分 10 分)

$$\text{解:} x - 5 + x^2 - 1 = 3(x - 1)$$

3 分

$$x^2 - 2x - 3 = 0$$

3 分

$$(x-3)(x+1) = 0$$

$$\text{解得 } x_1 = 3, x_2 = -1$$

2 分

经检验, $x = -1$ 是增根, 舍去,

1 分

$\therefore$ 原方程的解为 $x = 3$

1 分

21. (本题满分 10 分, 第(1)小题 5 分, 第(2)小题 5 分)

解(1) 在 $\text{Rt}\triangle BDE$ 中, $DE \perp AB$, $BD = 3\sqrt{2}$, $\angle ABC = 45^\circ$,

$$\therefore BE = DE = 3,$$

2 分

在 $\text{Rt}\triangle ADE$ 中, $\sin \angle DAB = \frac{3}{5}$, $DE = 3$,

$$\therefore AE = 4,$$

2 分

$$\therefore AB = AE + BE = 4 + 3 = 7$$

1 分

(2) 作 $CH \perp AB$, 垂足为 H

1 分

$\because AD$ 是 BC 边上的中线, $DB = 3\sqrt{2}$,

$$\therefore BC = 6\sqrt{2},$$

1 分

$\because \angle ABC = 45^\circ$, $\therefore BH = CH = 6$,

1 分

$$\therefore AH = 7 - 6 = 1$$

1 分

即在 $\text{Rt}\triangle CHA$ 中, $\cot \angle CAB = \frac{AH}{CH} = \frac{1}{6}$

1 分

22. (本题满分 10 分, 第(1)小题 4 分, 第(2)小题 6 分)

解:(1) 设 $y_{\text{甲}} = kx (k \neq 0)$

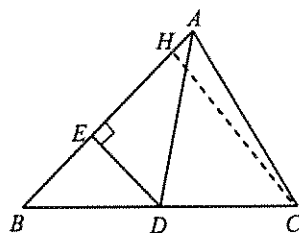
1 分

则 $0.5k = 7.5$, $\therefore k = 15$,

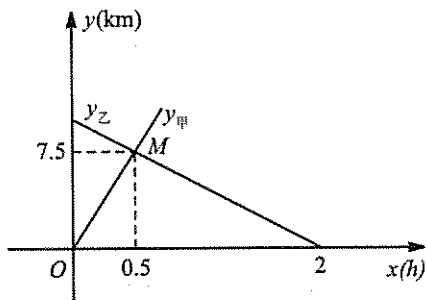
2 分

$$\therefore y_{\text{甲}} = 15x.$$

1 分



(第 21 题图)



(第 22 题图)

(2) 解法一:

$$\text{设 } y_{\text{甲}} = kx + b (k \neq 0)$$

1 分

把点(1.5, 7.5)、(2, 0)分别代入, 得:

$$\begin{cases} 7.5 = 0.5k + b \\ 0 = 2k + b \end{cases}$$

2 分

$$\text{解得 } \begin{cases} k = -5 \\ b = 10 \end{cases}$$

$$\therefore y_{\text{乙}} = -5x + 10$$

2 分

$$\therefore AB = 5 \times 2 = 10 \text{ km.}$$

1 分

解法二:

设乙的速度为 v km/h,

1 分

$$\text{则 } 2v = 0.5v + 7.5$$

2 分

$$\therefore v = 5$$

1 分

$$\therefore AB = 5 \times 2 = 10 \text{ km.}$$

2 分

23. (本题满分 12 分, 第(1)小题 4 分, 第(2)小题 4 分, 第(3)小题 4 分)

解: (1) $\because AD \parallel BC, BC = 2AD,$

点 E 是 BC 上的中点, $\therefore BC = 2CE$

1 分

$$\therefore AD = CE,$$

2 分

$\therefore$ 四边形 $AECD$ 为平行四边形.

1 分

(2) $\because$ 四边形 $AECD$ 是平行四边形

$$\therefore \angle D = \angle AEC,$$

2 分

$$\text{又 } \angle EAF = \angle CAD, \therefore \angle EAC = \angle DAF,$$

1 分

$$\therefore \triangle AEC \sim \triangle ADF$$

1 分

(3) 设 $AD = a$, 则 $BC = 2a$,

$$\text{又 } \because \angle ECA = 45^\circ, \angle B = 90^\circ,$$

$$\therefore AB = BC = 2a, AE = DC = \sqrt{5}a$$

$$\therefore \triangle AEC \sim \triangle ADF$$

$$\therefore \frac{AE}{AD} = \frac{EC}{DF}, \text{ 即 } \frac{\sqrt{5}a}{a} = \frac{a}{DF}, \therefore DF = \frac{\sqrt{5}}{5}a,$$

1 分

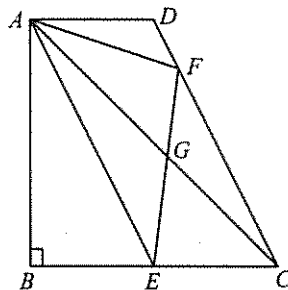
$$\therefore CF = DC - DF = \sqrt{5}a - \frac{\sqrt{5}}{5}a = \frac{4\sqrt{5}}{5}a.$$

1 分

$$\therefore AE \parallel DC$$

$$\therefore \frac{FG}{EG} = \frac{FC}{AE} = \frac{\frac{4\sqrt{5}}{5}a}{\sqrt{5}a} = \frac{4}{5}.$$

2 分



(第 23 题图)

24. (本题满分 12 分, 第(1)小题 6 分, 第(2)小题 6 分)

解: (1) $\because$ 矩形 $OMPN, OM = 6, ON = 3$

$$\therefore \text{点 } P(6, 3)$$

$\because$ 点 C, D 都在反比例函数 $y = \frac{6}{x}$ 图像上,

且点C在PN上,点D在PM上,

$\therefore$ 点C(2,3),点D(6,1)

又 $DB \perp y$ 轴, $CA \perp x$ 轴,

$\therefore A(2,0), B(0,1)$

$\therefore BG=2, GD=4, CG=2, AG=1$

$$\therefore \frac{AG}{GC} = \frac{1}{2}, \frac{BG}{GD} = \frac{2}{4} = \frac{1}{2}$$

$$\therefore \frac{AG}{GC} = \frac{BG}{GD}$$

$\therefore AB \parallel CD$.

又解:求直线CD的解析式为 $y = -\frac{1}{2}x + 4$, 直线

AB的解析式为 $y = -\frac{1}{2}x + 1$.

因为两直线的斜率相等,在y轴上的截距不等,所以两直线平行.(酌情给分)

(2) ① $\because PN \parallel DB$

$\therefore$ 当 $DE_1 = BC$ 时,四边形 BCE_1D 是等腰梯形

此时 $Rt\triangle CNB \cong Rt\triangle E_1PD$,

$\therefore PE_1 = CN = 2$,

$\therefore$ 点 $E_1(4,3)$

② $\because CD \parallel AB$,

当 E_2 在直线AB上, $DE_2 = BC = 2\sqrt{2}$,

四边形 $BCDE_2$ 为等腰梯形,

直线AB的解析式为 $y = -\frac{1}{2}x + 1$

$\therefore$ 设点 $E_2(x, -\frac{1}{2}x + 1)$

$$DE_2 = BC = 2\sqrt{2},$$

$$\therefore (x-6)^2 + (\frac{1}{2}x)^2 = 8$$

$$x_1 = \frac{28}{5}, x_2 = 4(\text{舍去})$$

$$\therefore E_2(\frac{28}{5}, -\frac{9}{5});$$

25. (本题满分14分,第(1)小题4分,第(2)小题4分,第(3)小题6分)

解:(1) 作 $AG \perp BC$ 于G, $BH \perp AC$ 于H,

$\because AB = AC, AG \perp BC, \therefore BG = GC = 2$,

$$\therefore AG = \sqrt{AC^2 - CG^2} = \sqrt{6^2 - 2^2} = 4\sqrt{2}$$

又 $AG \cdot BC = BH \cdot AC$,

$$\therefore BH = \frac{AG \cdot BC}{AC} = \frac{4\sqrt{2} \times 4}{6} = \frac{8\sqrt{2}}{3}$$

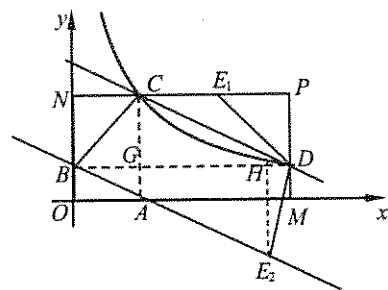
2分

2分

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1分

(第24题图)

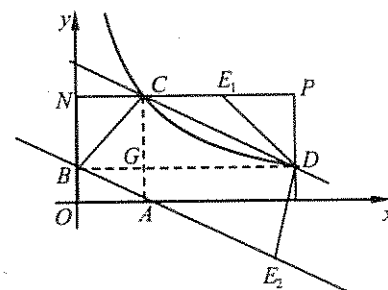


(第24题图)

2分

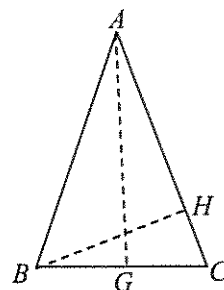
1分

(第24题图)



1分

2分



1分

1分

1分

1分

∴当⊙B与直线AC相切时, $x = \frac{8}{3}\sqrt{2}$.

(2) 作 $DF \perp BC$ 于 F ,

则 $DF \parallel AG$, ∴ $\frac{BD}{AB} = \frac{DF}{AG}$,

即 $\frac{x}{6} = \frac{DF}{4\sqrt{2}}$, ∴ $DF = \frac{2\sqrt{2}}{3}x$

$BF = BD \cdot \sin B = \frac{1}{3}x$,

∴ $CF = 4 - \frac{1}{3}x$,

在 $Rt\triangle CFD$ 中, $CD^2 = DF^2 + CF^2$

∴ $y = \sqrt{\left(4 - \frac{1}{3}x\right)^2 + \left(\frac{2\sqrt{2}}{3}x\right)^2}$

$= \sqrt{x^2 - \frac{8}{3}x + 16}$

($0 < x \leq 4$).

(3) 解法一:

① 作 $PQ \perp BC$ 于 Q .

∵ EF 是⊙B、⊙P的公共弦,

∴ $BP \perp EF$, 且 $EG = FG$,

∵ ⊙P 经过点 E , ∴ $PA = PE = PC$,

∴ $AE \perp BC$,

又 $AC = AB$, ∴ $BE = EC = 2$

∵ $PQ \parallel AE$, 且 P 是 AC 的中点

∴ $PQ = \frac{1}{2}AE = 2\sqrt{2}$, $CP = 3$,

∴ $CQ = 1$, $BQ = 3$,

∴ $BP = \sqrt{17}$

设 BP 交 EF 于点 H

设 $BH = m$, 由 $BE^2 - BH^2 = PE^2 - PH^2$,

$2^2 - m^2 = 3^2 - (\sqrt{17} - m)^2$

解得 $m = \frac{4}{17}\sqrt{34}$,

∴ $EF = 2m = \frac{8}{17}\sqrt{34}$

解法二:

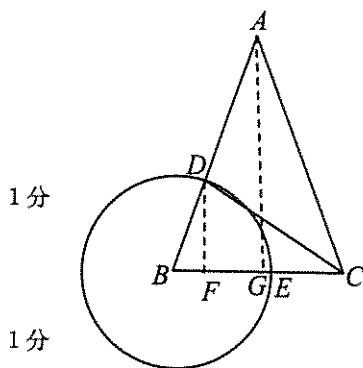
作 $PQ \perp BC$ 于 Q .

∵ EF 是⊙B、⊙P的公共弦,

∴ $BP \perp EF$, 且 $EG = FG$,

∵ ⊙P 经过点 E , ∴ $PA = PE = PC$,

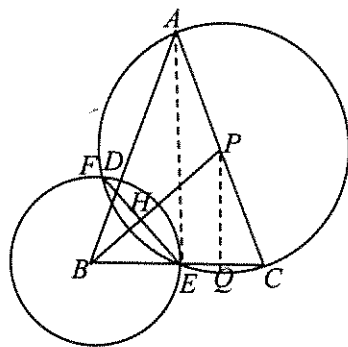
∴ $AE \perp BC$,



1分

1分

1分



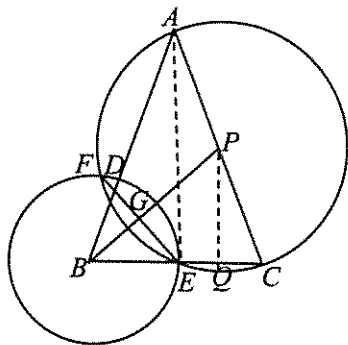
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又 $AC=AB$, $\therefore BE=EC=2$

$\because PQ \parallel AE$, 且 P 是 AC 的中点,

$$\therefore PQ = \frac{1}{2}AE = 2\sqrt{2}, CP = 3,$$

$$\therefore CQ = 1, BQ = 3,$$

$$\therefore BP = \sqrt{17}$$

1 分

而 $Rt\triangle BQP \sim Rt\triangle BGE$,

1 分

$$\therefore \frac{EG}{PQ} = \frac{BE}{BP}, \text{ 即 } \frac{EG}{2\sqrt{2}} = \frac{2}{\sqrt{17}}, \therefore EG = \frac{4\sqrt{34}}{17}$$

1 分

$$\therefore \text{公共弦 } EF = \frac{8\sqrt{34}}{17}$$

1 分

$$\textcircled{2} \text{ 当点 } E \text{ 和点 } C \text{ 重合时, } EF = \frac{16}{17}\sqrt{34}$$

2 分

长宁(金山)区中考数学质量抽查试卷·参考答案

一、选择题(本大题共 6 题,每题 4 分,满分 24 分)

1. C 2. B 3. C 4. C 5. D 6. B

二、填空题(本大题共 12 题,每题 4 分,满分 48 分)

7. $\frac{1}{9}$ 8. $(x+3y)(x-3y)$ 9. $x=1$ 10. $x \neq 2$ 11. $y = -x + 5$ 12. 1 13. 14 14. $\frac{1}{2}\vec{m} + \frac{1}{2}\vec{n}$

15. 8 16. $\frac{4}{7}$ 17. 15 或者 105 18. $\frac{8\sqrt{10}}{5}$

三、解答题(本大题共 7 题,满分 78 分)

19. (本题满分 10 分)

$$\text{原式} = \left(\frac{\sqrt{2}}{2}\right)^2 + 1 - \frac{2\sqrt{3}}{\sqrt{3}-1} + \sqrt{3} \quad 5 \text{ 分}$$

$$= \frac{1}{2} + 1 - \frac{2\sqrt{3}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} + \sqrt{3} \quad 2 \text{ 分}$$

$$= \frac{1}{2} + 1 - 3 - \sqrt{3} + \sqrt{3} \quad 1 \text{ 分}$$

$$= -\frac{3}{2} \quad 2 \text{ 分}$$

20. (本题满分 10 分)

$$\text{方法 1: 解: } \begin{cases} x-2y=3 & \textcircled{1} \\ x^2+xy-2y^2=0 & \textcircled{2} \end{cases}$$

$$\text{由方程}\textcircled{1}, \text{得: } x=3+2y \quad \textcircled{3} \quad 1 \text{ 分}$$

$$\text{把}\textcircled{3} \text{代入}\textcircled{2}, \text{得: } (3+2y)^2 + (3+2y)y - 2y^2 = 0 \quad 1 \text{ 分}$$

$$\text{整理,得: } 4y^2 + 15y + 9 = 0 \quad 2 \text{ 分}$$

$$\text{解这个方程,得: } y_1 = -\frac{3}{4}, y_2 = -3 \quad 2 \text{ 分}$$

$$\text{把 } y_1 = -\frac{3}{4}, y_2 = -3 \text{ 代入}\textcircled{3}, \text{得: } x_1 = \frac{3}{2}, x_2 = -3 \quad 2 \text{ 分}$$

$$\text{原方程组的解是: } \begin{cases} x_1 = \frac{3}{2} \\ y_1 = -\frac{3}{4} \end{cases}, \begin{cases} x_2 = -3 \\ y_2 = -3 \end{cases} \quad 2 \text{ 分}$$

$$\text{方法 2: 解: } \begin{cases} x-2y=3 & \text{①} \\ x^2+xy-2y^2=0 & \text{②} \end{cases}$$

由方程②, 得: $x+2y=0$ 或者 $x-y=0$ 2 分

$$\text{原方程可以化成两个方程组: } \begin{cases} x-2y=3 \\ x+2y=0 \end{cases} \text{ 和 } \begin{cases} x-2y=3 \\ x-y=0 \end{cases} \quad 2 \text{ 分}$$

$$\text{分别解这两个方程组, 得原方程组的解是: } \begin{cases} x_1 = \frac{3}{2} \\ y_1 = -\frac{3}{4} \end{cases}, \begin{cases} x_2 = -3 \\ y_2 = -3 \end{cases} \quad 6 \text{ 分}$$

21. (本题满分 10 分, 每小题 5 分)

解: (1) 设直线 OP 和直线 AQ 的解析式分别为 $y=k_1x$ 和 $y=k_2x+b_2$.

根据题意, 得: 点 Q 的坐标为 $(1, -m)$ 1 分

$$k_1 = m, \begin{cases} k_2 + b_2 = -m \\ 2k_2 + b_2 = 0 \end{cases}, \quad 2 \text{ 分}$$

$$\text{解得: } \begin{cases} k_2 = m \\ b_2 = -2m \end{cases} \quad 1 \text{ 分}$$

$\therefore k_1 = k_2 = m, \therefore$ 直线 $OP \parallel$ 直线 AQ 1 分

(2) $\because OP \parallel AQ, PB \parallel OA,$

$\therefore$ 四边形 $POAQ$ 是平行四边形

$\because AP \perp BO$

$\therefore$ 四边形 $POAQ$ 是菱形, 1 分

$\therefore PO = AO,$ 1 分

$$\therefore \sqrt{1+m^2} = 2, m = \pm\sqrt{3}. \quad 1 \text{ 分}$$

$\because m > 0, \therefore m = \sqrt{3}, \therefore$ 点 P 的坐标是 $(1, \sqrt{3})$. 2 分

22. (本题满分 10 分, 每小题 5 分)

解: (1) 联结 AD .

设 $BD = 2k, CD = \sqrt{3}k$. 1 分

$\because DE$ 垂直平分 $AB, \therefore AD = BD = 2k$. 1 分

$$\text{在 Rt}\triangle ACD \text{ 中, } \angle C = 90^\circ, \therefore \cos \angle ADC = \frac{CD}{AD} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}, \quad 2 \text{ 分}$$

$\therefore \angle ADC = 30^\circ$. 1 分

(2) $\because AD = BD, \therefore \angle B = \angle DAB$

$\because \angle ADC = 30^\circ, \angle B + \angle DAB = \angle ADC, \therefore \angle B = \angle DAB = 15^\circ$. 1 分

在 $\text{Rt}\triangle ACD$ 中, $\angle C = 90^\circ, \therefore AC = \sqrt{AD^2 - CD^2} = k$, 1 分

在 $\text{Rt}\triangle ABC$ 中, $\angle C = 90^\circ, \therefore \tan B = \frac{AC}{BC} = \frac{k}{\sqrt{3}k + 2k} = 2 - \sqrt{3}$, 2 分

$$\therefore \tan 15^\circ = 2 - \sqrt{3}.$$

1分

23. (本题满分12分, 每小题6分)

证明: (1) $\because BD$ 是 $\triangle ABC$ 的角平分线, $\therefore \angle ABD = \angle CBD$.

$$\because DE \parallel AB, \therefore \angle ABD = \angle BDE, \therefore \angle CBD = \angle BDE,$$

$$\therefore DE = BE.$$

2分

$$\because DE \parallel AB, \therefore \angle DEF = \angle BFE,$$

$$\because \angle DEF = \angle A, \therefore \angle A = \angle BFE, \therefore AD \parallel EF,$$

$$\therefore \text{四边形 } ADEF \text{ 是平行四边形}, \therefore AF = DE,$$

2分

$$\therefore BE = AF.$$

2分

$$(2) \because DE \parallel AB, \therefore \frac{BN}{ND} = \frac{AB}{DE}.$$

2分

$$\because EF \parallel AC, \therefore \frac{BD}{MD} = \frac{AB}{AF}.$$

2分

$$\because AF = DE, \therefore \frac{BN}{ND} = \frac{BD}{MD}, \therefore BN \cdot MD = BD \cdot ND.$$

2分

24. (本题满分12分, 每小题4分)

$$\text{解: (1) 根据题意, 得 } \begin{cases} 1+b+c=0 \\ c=3 \end{cases},$$

1分

$$\therefore b = -4, c = 3.$$

1分

$$\therefore \text{抛物线解析式为 } y = x^2 - 4x + 3.$$

1分

$$\text{顶点 } P \text{ 的坐标是 } (2, -1).$$

1分

(2) $\because$ 顶点 $P(2, -1)$ $\therefore$ 抛物线对称轴是直线 $x = 2$

$$\because \text{抛物线 } y = x^2 + bx + c \text{ 与 } x \text{ 轴相交于点 } A \text{ 和点 } B,$$

$$\therefore \text{点 } A \text{ 和点 } B \text{ 关于对称轴直线 } x = 2 \text{ 对称}$$

$$\because \text{点 } A(1, 0) \therefore \text{点 } B \text{ 的坐标是 } (3, 0).$$

$$\text{设直线 } PB \text{ 的解析式是 } y = kx + b,$$

$$\text{根据题意, 得: } \begin{cases} 2k + b = -1 \\ 3k + b = 0 \end{cases}, \text{ 解得: } k = 1, b = -3.$$

$$\therefore \text{直线 } PB \text{ 的解析式为 } y = x - 3.$$

1分

$$\therefore \text{点 } D \text{ 的坐标为 } (t, t^2 - 4t + 3), \text{ 点 } E \text{ 的坐标为 } (t, t - 3).$$

1分

$$DE = t^2 - 5t + 6, EF = t - 3,$$

$$\therefore t^2 - 5t + 6 = 2(t - 3), \text{ 解得: } t_1 = 3, t_2 = 4.$$

1分

$$\because t > 3, \therefore t = 4$$

$$\therefore \text{点 } D \text{ 的坐标为 } (4, 3)$$

1分

(3) 证明: 由(2)得: 点 E 的坐标为 $(4, 1)$,

$$\therefore DE = 2, BE = \sqrt{2}, PE = 2\sqrt{2}, \therefore \frac{DE}{PE} = \frac{BE}{DE}.$$

2分

$$\because \angle DEB = \angle PED, \therefore \triangle BDE \sim \triangle DPE, \therefore \angle BDE = \angle DPE.$$

2分

25. (本题满分14分, 第(1)小题4分, 第(2)小题4分, 第(3)小题6分)

$$\text{解: (1) 在 } \text{Rt}\triangle ABC \text{ 中, } \angle ACB = 90^\circ, BC = AB \cdot \sin A = 4, \therefore AC = 3,$$

1分

$$\therefore PC = PQ, \therefore \angle PCQ = \angle PQC.$$

1分

$$\because \angle QED = 90^\circ, \therefore \angle QDE + \angle PQC = 90^\circ,$$

$$\because \angle PCQ + \angle ACD = 90^\circ, \therefore \angle QDE = \angle ACD.$$

$$\because \angle QDE = \angle ADC, \therefore \angle ADC = \angle ACD, \therefore AD = AC = 3.$$

2分

(2) 作 $QH \perp BC$, 垂足为点 H .

$$\because \angle PEB = \angle ACB = 90^\circ, \therefore \angle BPE + \angle ABC = 90^\circ, \angle ABC + \angle A = 90^\circ,$$

$$\therefore \angle QPH = \angle A, \therefore \sin \angle QPH = \frac{4}{5}.$$

1分

$$\because PQ = PC = x, \therefore QH = \frac{4}{5}x,$$

1分

$$\therefore y = \frac{1}{2} \cdot \frac{4}{5}x \cdot x, \text{ 即 } y = \frac{2}{5}x^2,$$

1分

$$\text{定义域为 } \frac{3}{2} \leq x \leq 4.$$

1分

(3) 解法一:

$$\text{在 Rt}\triangle PBE \text{ 中, } \angle PEB = 90^\circ, BP = 4 - x, \sin \angle BPE = \frac{4}{5},$$

$$\therefore BE = \frac{4}{5}(4 - x) = \frac{16}{5} - \frac{4}{5}x, PE = \frac{3}{5}(4 - x) = \frac{12}{5} - \frac{3}{5}x,$$

1分

$$\therefore EF = \frac{4}{5}x, EQ = \frac{8}{5}x - \frac{12}{5}.$$

1分

$$\therefore PF = \sqrt{\left(\frac{12}{5} - \frac{3}{5}x\right)^2 + \left(\frac{4}{5}x\right)^2} = \sqrt{x^2 - \frac{72}{25}x + \frac{144}{25}},$$

$$QF = \sqrt{\left(\frac{8}{5}x - \frac{12}{5}\right)^2 + \left(\frac{4}{5}x\right)^2} = \sqrt{\frac{16}{5}x^2 - \frac{192}{25}x + \frac{144}{25}}.$$

1分

$$\text{如果 } PF = PQ, \text{ 那么 } \sqrt{x^2 - \frac{72}{25}x + \frac{144}{25}} = x, \text{ 解得 } x = 2.$$

1分

$$\text{如果 } PF = QF, \text{ 那么 } \sqrt{x^2 - \frac{72}{25}x + \frac{144}{25}} = \sqrt{\frac{16}{5}x^2 - \frac{192}{25}x + \frac{144}{25}},$$

$$\text{解得 } x_1 = 0 (\text{不合题意, 舍去}), x_2 = \frac{24}{11}.$$

1分

综上所述, 如果 $\triangle PQF$ 是以 PF 为腰的等腰三角形, CP 的长为 2 或 $\frac{24}{11}$.

1分

$$\text{解法二: 在 Rt}\triangle PBE \text{ 中, } \angle PEB = 90^\circ, BP = 4 - x, \sin \angle BPE = \frac{4}{5},$$

$$\therefore BE = \frac{4}{5}(4 - x) = \frac{16}{5} - \frac{4}{5}x, PE = \frac{3}{5}(4 - x) = \frac{12}{5} - \frac{3}{5}x$$

$$\therefore EF = \frac{4}{5}x, EQ = \frac{8}{5}x - \frac{12}{5}.$$

1分

$$\text{如果 } PF = PQ, \text{ 那么 } PF = PC, \therefore \angle PCF = \angle PFC, \angle B = \angle PFB,$$

$$\therefore PF = PB, \therefore CP = PB = 2.$$

2分

$$\text{如果 } PF = FQ, \text{ 那么 } PE = EQ, \therefore \frac{12}{5} - \frac{3}{5}x = \frac{8}{5}x - \frac{12}{5},$$

$$\text{解得 } x = \frac{24}{11}, \therefore CP = \frac{24}{11}.$$

2分

综上所述, 如果 $\triangle PQF$ 是以 PF 为腰的等腰三角形, CP 的长为 2 或 $\frac{24}{11}$.

1分

静安(青浦)区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共6题,每题4分,满分24分)

1. A 2. D 3. B 4. C 5. B 6. D

二、填空题:(本大题共12题,满分48分)

7. $-\frac{1}{8}$ 8. 2 9. $x=3$ 10. $x \leq \frac{3}{2}$ 11. $m < 10$ 12. $y = ax^2 + bx + c (a < 0, b > 0)$ 或 $y = a(x+m)^2 + h (a < 0, m < 0)$ 13. 甲 14. $\frac{3}{5}$ 15. 135° 16. $\frac{1}{2}\vec{a} + \vec{b}$ 17. $0 \leq d < 1$ 或 $d > 5$ 18. $\sqrt{6}$

三、(本大题共7题,第19~22题每题10分,第23、24题每题12分,第25题14分,满分78分)

$$19. \text{解: 原式} = \frac{(a-b)^2}{(a+b)(a-b)} \div \frac{a-b}{ab} \quad (3 \text{分})$$

$$= \frac{(a-b)^2}{(a+b)(a-b)} \cdot \frac{ab}{a-b} = \frac{ab}{a+b} \quad (1+2 \text{分})$$

$$\text{当 } a = \sqrt{5} + 1, b = \sqrt{5} - 1 \text{ 时, 原式} = \frac{(\sqrt{5}+1)(\sqrt{5}-1)}{\sqrt{5}+1+(\sqrt{5}-1)} = \frac{4}{2\sqrt{5}} = \frac{2}{5}\sqrt{5}. \quad (1+2+1 \text{分})$$

20. 解: $\because$ 双曲线 $y = \frac{k}{x}$ 经过点 $A(a, a+4)$ 和点 $B(2a, 2a-1)$,

$$\therefore \begin{cases} a+4 = \frac{k}{a}, \\ 2a-1 = \frac{k}{2a}, \end{cases} \quad (4 \text{分})$$

$$\therefore \begin{cases} a(a+4) = k, \\ 2a(2a-1) = k, \end{cases} \quad (2 \text{分})$$

$$\therefore a^2 + 4a = 4a^2 - 2a, \quad (1 \text{分})$$

$$\therefore a = 0 \text{ 或 } a = 2. \quad (1 \text{分})$$

其中 $a = 0$ (不符合题意). (1分)

$$\therefore k = 12. \quad (1 \text{分})$$

21. 解:(1) 在 $\text{Rt}\triangle ABC$ 中, $\cos \angle ABC = \frac{AB}{BC}$, (1分)

$$\therefore AB = BC \cdot \cos \angle ABC = 5 \times \frac{\sqrt{5}}{5} = \sqrt{5}, \quad (2 \text{分})$$

$$\therefore AC = \sqrt{BC^2 - AB^2} = \sqrt{5^2 - (\sqrt{5})^2} = 2\sqrt{5}. \quad (2 \text{分})$$

(2) 过点 B 作 $BE \perp AD$, 垂足为 E , (1分)

$\because AD \parallel BC, \therefore \angle EAB = \angle ABC$, (1分)

$$\therefore \frac{AE}{AB} = \cos \angle EAB = \cos \angle ABC = \frac{\sqrt{5}}{5}, \therefore AE = 1. \quad (1 \text{分})$$

$$\therefore BE = \sqrt{AB^2 - AE^2} = \sqrt{(\sqrt{5})^2 - 1^2} = 2. \quad (1 \text{分})$$

$$\therefore DE = AE + AD = 1 + 2 = 3, \therefore \tan \angle ADB = \frac{BE}{DE} = \frac{2}{3}. \quad (1 \text{分})$$

22. 解:(1) 设乙种树木有 x 棵, 则甲种树木有 $(2x-600)$ 棵, (1分)

$$\text{得 } 2x - 60 + x = 6600, \quad (2 \text{分})$$

解得 $x = 2400$, 则 $2x - 600 = 4200$. (1分)

(2) 设安排 y 人种甲种树木, $(26 - y)$ 人种乙种树木, (1分)

$$\text{得 } \frac{4200}{60y} = \frac{2400}{40(26 - y)}, \quad (2分)$$

解得 $y = 14$, (1分)

经检验它是方程的解, 符合题意.

则 $26 - y = 12$. (1分)

答: 甲种树木有 4200 棵, 则乙种树木有 2400 棵; 应安排 14 人种甲种树木, 12 人种乙种树木.

(1分)

23. (1) 证明: $\because$ 四边形 $ABCD$ 是菱形, $\therefore AD \parallel BC$. (1分)

$$\therefore \angle ADC = \angle DCF. \quad (1分)$$

$$\text{又 } \because AD = CD, DE = CF, \quad (1分)$$

$$\therefore \triangle ADE \cong \triangle DCF. \quad (1分)$$

$$\therefore \angle CDF = \angle DAE. \quad (1分)$$

(2) 证明: $\because \angle GDE = \angle GAD, \angle EGD = \angle DGF, \therefore \triangle GDE \sim \triangle GAD$. (2分)

$$\because CF = DE, CD = AD, \therefore \frac{DE}{AD} = \frac{DE}{CD} = \frac{1}{2}. \quad (1分)$$

$$\therefore \frac{GE}{GD} = \frac{GD}{AG} = \frac{DE}{AD} = \frac{1}{2}. \quad (2分)$$

$$\therefore AG = 2DG = 4EG. \quad (1分)$$

$$\therefore AE = 3EG. \quad (1分)$$

24. 解: (1) $\because$ 抛物线 $y = ax^2 + bx - 1$ 与 y 轴的交点为 $(0, 1)$, (1分)

点 $(0, -1)$ 与点 $A(2, -1)$ 是关于抛物线对称轴的对称点, (1分)

$\therefore$ 抛物线的对称轴为直线 $x = 1$, (1分)

$\therefore$ 点 B 的坐标为 $(1, 0)$. (1分)

$\because$ 抛物线 $y = ax^2 + bx - 1$ 经过点 $A(2, -1)$, $\therefore -1 = 4a + 2b - 1$, (1分)

$$\therefore b = -2a. \quad (1分)$$

$$\therefore \text{抛物线对称轴为直线 } x = -\frac{b}{2a} = -\frac{-2a}{2a} = 1, \quad (1分)$$

$\therefore$ 点 B 的坐标为 $(1, 0)$. (1分)

(2) $\because$ 直线 $y = x + 1$ 与此抛物线的对称轴交于点 C , $\therefore$ 点 $C(1, 2)$.

设直线 $y = x + 1$ 与 x 轴交于点 E , 当 $y = 0$ 时, $x = -1$, $\therefore$ 点 E 坐标为 $(-1, 0)$.

$$\therefore BC = BE = 2, \therefore \angle BCE = 45^\circ, \quad (1分)$$

过点 A 作 $AF \perp BC$, 垂足为 F , $AF = BF = 1$, $\therefore \angle ABF = 45^\circ, AB = \sqrt{2}$.

$$\therefore \angle BCD = \angle ABC = 135^\circ, \quad (1分)$$

$$\because \angle BDC = \angle ACB. \therefore \triangle BCD \sim \triangle ABC. \quad (1分)$$

$$\therefore \frac{CD}{BC} = \frac{BC}{AB}, \frac{CD}{2} = \frac{2}{\sqrt{2}}, \therefore CD = 2\sqrt{2}. \quad (1分)$$

过点 D 作 $DG \perp BC$, 垂足为 G , $\because \angle DCG = \angle BCE = 45^\circ$, $\therefore DG = CH = 2$.

$\therefore$ 点 $D(3, 4)$. (1分)

$$\because b = -2a, \therefore \text{抛物线为 } y = ax^2 - 2ax - 1. \quad (1分)$$

$$\therefore 4 = 9a - 6a - 1, \therefore a = \frac{5}{3}. \quad (1 \text{ 分})$$

$$\therefore \text{抛物线的表达式为 } y = \frac{5}{3}x^2 - \frac{10}{3}x - 1. \quad (1 \text{ 分})$$

25. 解: (1) 联结 OD , 在 $\odot O$ 中, $\because OC \perp DE, OC = OA - AC = 4, BC = AB - AC = 9$.

$$\therefore CE = CD = \sqrt{OD^2 - OC^2} = \sqrt{5^2 - 4^2} = 3. \quad (2 \text{ 分})$$

$$BE = \sqrt{BC^2 + CE^2} = \sqrt{9^2 + 3^2} = 3\sqrt{10}. \quad (2 \text{ 分})$$

(2) 与(1)同理得:

$$CE = CD = \sqrt{OD^2 - OC^2} = \sqrt{5^2 - (5-x)^2} = \sqrt{10x - x^2}. \quad (1 \text{ 分})$$

$$BE = \sqrt{BC^2 + CE^2} = \sqrt{(10-x)^2 + 10x - x^2} = \sqrt{100 - 10x}. \quad (1 \text{ 分})$$

$$\because DF \perp EB, \therefore \cos E = \frac{EF}{DE} = \frac{CE}{BE}. \quad (1 \text{ 分})$$

$$\therefore \frac{y}{2\sqrt{10x-x^2}} = \frac{\sqrt{10x-x^2}}{\sqrt{100-10x}}, \quad (1 \text{ 分})$$

$$\therefore y \text{ 与 } x \text{ 之间的函数解析式为 } y = \frac{x\sqrt{100-10x}}{5}, \quad (1 \text{ 分})$$

$$\text{定义域为 } 0 < x \leq 5. \quad (1 \text{ 分})$$

(3) 当点 F 在线段 EB 上时,

$$\because EF = 3BF, \therefore EF = \frac{3}{4}BE, \text{ 得 } \frac{x}{5}\sqrt{100-10x} = \frac{3}{4}\sqrt{100-10x}, \quad (1 \text{ 分})$$

$$\text{解得 } x_1 = 10 (\text{不符合题意}), x_2 = \frac{15}{4}. \quad (1 \text{ 分})$$

$$\text{当点 } F \text{ 在线段 } EB \text{ 延长线上时, 同理 } BE = \sqrt{100-10x}, EF = \frac{x\sqrt{100-10x}}{5}.$$

$$\because EF = 3BF, \therefore EF = \frac{3}{2}BE, \text{ 得 } \frac{x}{5}\sqrt{100-10x} = \frac{3}{2}\sqrt{100-10x}, \quad (1 \text{ 分})$$

$$\text{解得 } x_1 = 10 (\text{不符合题意}), x_2 = \frac{15}{2}. \quad (1 \text{ 分})$$

$$\therefore \text{线段 } AC \text{ 的长为 } \frac{15}{4} \text{ 或 } \frac{15}{2}.$$

宝山(嘉定)区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共 6 题,每题 4 分,满分 24 分)

1. C 2. C 3. D 4. B 5. D 6. B

二、填空题:(本大题共 12 题,每题 4 分,满分 48 分)

7. 3.12×10^6 8. $2(x-2)(x+2)$ 9. $1 < x < 2$ 10. $k > 1$ 11. $y = x^2 + 2$ 12. 乙 13. $x = -1$

14. $\frac{1}{2}a + \frac{1}{2}b$ 15. $\frac{3}{5}$ 16. $20\sqrt{3} + 20$ 17. $a = 0$ 18. $\frac{3}{7}\sqrt{3}$

三、解答题:(本大题共 7 题,满分 78 分)

$$19. \text{ 解: } \left(\frac{\sqrt{x} - \frac{x}{x+\sqrt{x}}}{\frac{x-\sqrt{x}}{\sqrt{x}}} \right) \div \frac{x-\sqrt{x}}{\sqrt{x}} = \left(\sqrt{x} - \frac{\sqrt{x}}{\sqrt{x}+1} \right) \cdot \frac{1}{\sqrt{x}-1}$$

4 分

$$= \frac{x+\sqrt{x}-\sqrt{x}}{\sqrt{x}+1} \cdot \frac{1}{\sqrt{x}-1} \quad 2 \text{ 分}$$

$$= \frac{x}{x-1} \quad 2 \text{ 分}$$

当 $x=2+\sqrt{2}$ 时, 原式 $= \frac{2+\sqrt{2}}{\sqrt{2}+1} = \sqrt{2}$ 2 分

说明: 约分、二次根式的减法、除化乘、二次根式的乘法等每一步各 2 分,
代入(或约分或分母有利化方法不限)得出答案再加 2 分.

20. 解: 设 $\frac{2x-1}{x} = y$, 则原方程可化为: $y - \frac{3}{y} + 2 = 0$ 2 分

整理, 得: $y^2 + 2y - 3 = 0$ 2 分

解这个关于 y 的方程, 得 $y_1 = -3, y_2 = 1$. 2 分

当 $y = -3$ 时, 得方程 $\frac{2x-1}{x} = -3$, 解这个关于 x 的方程得 $x = \frac{1}{5}$. 1 分

当 $y = 1$ 时, 得方程 $\frac{2x-1}{x} = 1$, 解这个关于 x 的方程得 $x = 1$. 1 分

经检验: $x = \frac{1}{5}, x = 1$ 均为原方程的根 1 分

所以, 原方程的根是 $x_1 = \frac{1}{5}, x_2 = 1$ 1 分

方法 2: 方程两边同时乘以 $x(2x-1)$, 得 1 分

$$(2x-1)^2 - 3x^2 + 2x(2x-1) = 0$$
 3 分

整理, 得 $5x^2 - 6x + 1 = 0$ 2 分

解这个整式方程, 得 $x_1 = \frac{1}{5}, x_2 = 1$ 2 分

经检验: $x = \frac{1}{5}, x = 1$ 均为原方程的根 1 分

所以, 原方程的根是 $x_1 = \frac{1}{5}, x_2 = 1$ 1 分

21. 解: 根据题意易知 MN 为 AB 的垂直平分线 $\therefore DB = DA$ 2 分

$\therefore \angle A = \angle DBA$. 1 分

$\because \angle A = 34^\circ, \therefore \angle DBA = \angle A = 34^\circ$, 1 分

$\therefore \angle CDB = \angle A + \angle DBA = 68^\circ$ 1 分

又 $\because BC = CD \therefore \angle CBD = \angle CDB = 68^\circ$. 1 分

$\therefore \angle ABC = \angle ABD + \angle CBD = 34^\circ + 68^\circ = 102^\circ$ 2 分

$\therefore \angle C = 180^\circ - \angle CDB - \angle CBD = 44^\circ$ 2 分

22. 解: (1) 过点 C 作 $CH \perp OB$, 垂足为 H , 1 分

又 $\because AB \perp OB, \therefore CH \parallel AB. \therefore \frac{OH}{OB} = \frac{CH}{AB} = \frac{OC}{OA}$. 1 分

由 $A(-4, 2)$ 得 $AB = 2, OB = 4$. 由 C 为边 AO 中点, 得 $\frac{OC}{OA} = \frac{1}{2}$.

$\therefore OH = 2, CH = 1, \therefore C(-2, 1)$, 1 分

将 $C(-2, 1)$ 代入 $y = \frac{k}{x}$, 得 $k = -2$. 1 分

∴过C的反比例函数的解析式： $y = -\frac{2}{x}$ 1分

(2) 由题意易得直线AB的表达式为 $x = -4$.

因为点D在直线 $x = -4$ 上,所以可设 $D(-4, y)$. 1分

将点 $D(-4, y)$ 代入 $y = -\frac{2}{x}$, 得 $y = \frac{1}{2}$. 所以 $D(-4, \frac{1}{2})$ 1分

设CD的表达式为 $y = kx + b$, 将 $C(-2, 1)$ 、 $D(-4, \frac{1}{2})$ 代入, 得

$$\begin{cases} -2k + b = 1 \\ -4k + b = \frac{1}{2} \end{cases}, \text{解得} \begin{cases} k = \frac{1}{4} \\ b = \frac{3}{2} \end{cases}. \quad 1 \text{分}$$

所以直线CD的表达式为 $y = \frac{1}{4}x + \frac{3}{2}$ 1分

将 $y = 0$ 代入 $y = \frac{1}{4}x + \frac{3}{2}$ 得 $x = -6$

∴直线CD和x轴的交点坐标为 $(-6, 0)$. 1分

23. 解:(1) ∵ $DE \perp BC$, $BF \perp CD$, ∴ $\angle DEC = \angle BEH = 90^\circ$, $\angle EDC = \angle EBH = 90^\circ - \angle C$. 2分

在 $\text{Rt}\triangle EDB$ 中, ∵ $\angle DBC = 45^\circ$, $\angle DEB = 90^\circ$, ∴ $\angle BDE = 90^\circ - \angle DBC = 45^\circ$.

∴ $\angle DBE = \angle EBD$. ∴ $DE = BE$. 1分

在 $\triangle BHE$ 与 $\triangle DCE$ 中, ∵ $\angle EHB = \angle ECD$, $\angle EDC = \angle EBH$, $DE = BE$

∴ $\triangle BHE \cong \triangle DCE$. 2分

∴ $CD = BH$. 1分

(2) ∵ 四边形ABCD为平行四边形, ∴ $AB = CD$, $\angle A = \angle C$, $AD \parallel BC$. 1分

∵ $\triangle BHE \cong \triangle DCE$, ∴ $\angle EHB = \angle C$. 又 ∵ $\angle A = \angle C$, ∴ $\angle EHB = \angle A$. 1分

∵ $AD \parallel BC$, ∴ $\angle AGB = \angle EBH$ 1分

∴ $\triangle ABG \sim \triangle HEB$, 1分

∴ $\frac{AG}{AB} = \frac{BH}{HE}$ 即: $AB \cdot BH = AG \cdot HE$ 1分

∴ $AB^2 = AG \cdot HE$. 即AB为AG和HE的比例中项. 1分

24. 解:(1) 由抛物线 $y = -x^2 + bx + 3$ 经过点 $A(-1, 0)$, 易知

$-(-1)^2 - b + 3 = 0$, 解得 $b = 2$. 1分

进而得到该抛物线的表达式为 $y = -x^2 + 2x + 3$, 其对称轴为直线 $x = 1$.

因为抛物线 $y = -x^2 + 2x + 3$ 与y轴的交点 $C(0, 3)$, 1分

由点D与点C关于直线 $x = 1$ 对称, 得点D的坐标为 $(2, 3)$. 1分

过点D作 $DH \perp AO$, 垂足为H.

在 $\text{Rt}\triangle ADH$ 中, 因为 $\angle AHD = 90^\circ$, $DH = 4 = AH$, 所以 $\angle DAH = 45^\circ$.

所以直线AD与x轴的正方向的夹角为 45° . 1分

(2) 设直线AD的表达式为 $y = kx + b$, 将 $D(2, 3)$ 、 $C(0, 3)$ 代入, 得

$$\begin{cases} 2k + b = 3, \\ -k + b = 0 \end{cases}, \text{解得} \begin{cases} k = 1, \\ b = 1. \end{cases} \text{所以直线AD的表达式为 } y = x + 1.$$

因为E是抛物线 $y = -x^2 + 2x + 3$ 上横坐标为 m 的一个动点,

故可得 $E(m, -m^2 + 2m + 3)$,

由点 F 在直线 $y = x + 1$ 上, 且 EF 平行于 x 轴,

故可得 $F(-m^2 + 2m + 2, -m^2 + 2m + 3)$

1 分

根据题意点 E 在点 F 的左侧, $EF = -m^2 + m + 2$

1 分

由 EF 平行于 x 轴, $\angle EFG = \angle DAO = 45^\circ$,

得 $\triangle EFG$ 为等腰直角三角形

1 分

$\triangle EFG$ 的周长 $l = EF + 2 \times \frac{\sqrt{2}}{2} EF = -(1 + \sqrt{2})m^2 + (1 + \sqrt{2})m + 2 + 2\sqrt{2}$,

其中 $-1 < m < 2$

1 分

(3) 易知该抛物线的顶点是 $M(1, 4)$

① 如图 1, 若 AP 为矩形 $AMPQ$ 的对角线, 过 M 作 $MN \perp y$ 轴, 过 Q 作 $QH \perp x$ 轴, 垂足分别为 N, H . 由 $MN \parallel OA$ 得 $\frac{NI}{OI} = \frac{MN}{OA} = 1$. 又 $OA = 1, ON = 4$, 所以 $OI = IN = 2$.

由 $\tan \angle PMN = \tan \angle MIN = \tan \angle AIO = \frac{1}{2}$. 可求 $PN = MN \cdot \tan \angle PMN = \frac{1}{2}$.

利用 $\triangle PMN \cong \triangle QAH$ 得到 $QH = PN = \frac{1}{2}, AH = MN = 1$, 得到 $Q_1(-2, \frac{1}{2})$.

1 分

② 如图 2, 若 AP 为矩形 $APQM$ 的一边, 类似①可得 $\tan \angle PAO = \tan \angle AIO = \frac{1}{2}$,

利用 $\triangle QMN \cong \triangle APO$ 得到 $MN = OP = \frac{1}{2}, QN = AO = 1$, 得到 $Q_2(2, \frac{9}{4})$.

1 分

③ 如图 3, 若 AP 为矩形 $APMQ$ 的一边, 易得 $AI = \sqrt{OA^2 + OI^2} = \sqrt{5}$.

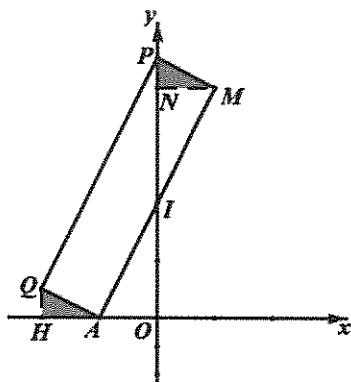
利用矩形的对角线相等易得 $PQ = MA = 2AI = 2\sqrt{5}, PI = IQ = IA = \sqrt{5}$.

$OQ = IQ - IO = \sqrt{5} - 2, PN = PI - IN = \sqrt{5} - 2$. 易得 $Q_3(0, 2 - \sqrt{5})$.

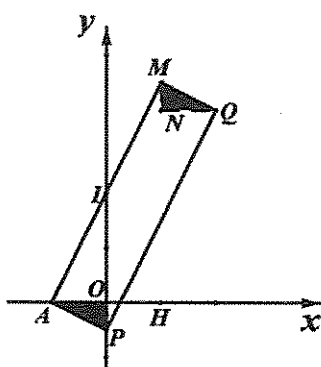
1 分

将图 3 中的字母 P 与 Q 对调, 可得 $Q_4(0, 2 + \sqrt{5})$.

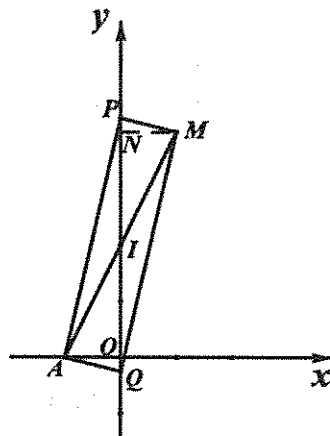
1 分



第 24 题图 1



第 24 题图 2



第 24 题图 3

25. 解: (1) 联结 OP 并延长交 AB 于 H , 如图 1.

$\because O, P$ 分别是 $\odot O, \odot P$ 的圆心, AB 是公共弦, $AB = 24$,

$\therefore OH \perp AB, AH = BH = \frac{1}{2} AB = 12$.

1 分

$$= 4 - \sqrt{3} \quad 2 \text{ 分}$$

20. 解: 由②得: $x - 2y = 0, x - y = 0$ 2 分

$$\text{原方程组可化为} \begin{cases} x + 2y = 1 \\ x - 2y = 0 \end{cases}, \begin{cases} x + 2y = 1 \\ x - y = 0 \end{cases} \quad 2 \text{ 分}$$

$$\text{解得原方程组的解为} \begin{cases} x = \frac{1}{2} \\ y = \frac{1}{4} \end{cases}, \begin{cases} x = \frac{1}{3} \\ y = \frac{1}{3} \end{cases} \quad 6 \text{ 分}$$

21. 解: (1) $\because$ 一次函数 $y = kx + b (k \neq 0)$ 的图像经过 $A(0, -2), B(1, 0)$ 两点

$$\therefore \begin{cases} k + b = 0 \\ b = -2 \end{cases} \text{ 解得 } \begin{cases} k = 2 \\ b = -2 \end{cases} \quad 1 \text{ 分}$$

$\therefore$ 一次函数的解析式为 $y = 2x - 2$ 1 分

设点 M 的坐标为 $(x, 2x - 2)$

$\because \triangle OBM$ 的面积是 2, M 在第一象限内

$$\therefore \frac{1}{2} \times 1 \times (2x - 2) = 2$$

$$x = 3 \quad 1 \text{ 分}$$

$$\therefore M(3, 4) \quad 1 \text{ 分}$$

$\because$ 点 $M(3, 4)$ 在反比例函数 $y = \frac{m}{x} (m \neq 0)$ 的图像上

$$\therefore m = 12$$

$$\therefore \text{反比例函数的解析式为 } y = \frac{12}{x} \quad 1 \text{ 分}$$

(2) $\because A(0, -2), B(1, 0), O(0, 0), M(3, 4)$

$$\therefore OB = 1$$

$$AB = \sqrt{1 + (-2)^2} = \sqrt{5} \quad 1 \text{ 分}$$

$$MB = \sqrt{(3-1)^2 + 4^2} = 2\sqrt{5} \quad 1 \text{ 分}$$

$$\because \angle AOB = \angle AMP = 90^\circ$$

$$\angle OBA = \angle MBP$$

$$\therefore \triangle OAB \sim \triangle MPB \quad 1 \text{ 分}$$

$$\therefore \frac{OB}{BM} = \frac{AB}{BP}$$

$$\therefore BP = 10 \quad 1 \text{ 分}$$

$$\therefore P(11, 0) \quad 1 \text{ 分}$$

22. (1) 100; $60 + 10t$ 各 2 分

(2) 过 O 作 $OH \perp PQ$, 垂足为 H

$$\therefore \angle OHP = 90^\circ$$

$$\text{由题意得 } \angle OPH = 65^\circ - 20^\circ = 45^\circ \quad 1 \text{ 分}$$

$$\therefore \cos \angle OPH = \frac{PH}{OP} = \frac{\sqrt{2}}{2}$$

$$\sin \angle POH = \frac{OH}{OP} = \frac{\sqrt{2}}{2}$$

$$\because OP = 200(\text{千米})$$

$$\therefore OH = PH = 100\sqrt{2}(\text{千米}) \quad 2 \text{分}$$

$$\therefore t = 100\sqrt{2} \div 20 = 5\sqrt{2}(\text{小时}) \quad 1 \text{分}$$

$$\text{此时半径为 } 60 + 10 \times 5\sqrt{2} \approx 130.5(\text{千米}) \quad 1 \text{分}$$

$$\therefore \text{台风中心与城市的最短距离为 } 100\sqrt{2} \approx 141(\text{千米})$$

$$\text{又} \because 141 > 130.5$$

$$\therefore \text{这股台风不会侵袭这座海滨城市.} \quad 1 \text{分}$$

23. (1) $\because$ 四边形 $ABCD$ 是正方形,

$$\therefore OA = \frac{1}{2}AC, OB = \frac{1}{2}BD, AC = BD, \angle AOE = \angle BOG = 90^\circ \quad 1 \text{分}$$

$$\therefore OA = OB \quad 1 \text{分}$$

$$\because BH \perp AF, \therefore \angle AHG = 90^\circ,$$

$$\therefore \angle GAH + \angle AGH = 90^\circ \quad 1 \text{分}$$

$$\because \angle BOG = 90^\circ$$

$$\therefore \angle OBG + \angle AGH = 90^\circ,$$

$$\therefore \angle GAH = \angle OBG \quad 1 \text{分}$$

$$\therefore \triangle OAE \cong \triangle OBG(\text{ASA}) \quad 1 \text{分}$$

$$\therefore AE = BG \quad 1 \text{分}$$

(2) $\because \triangle OAE \cong \triangle OBG$

$$\therefore OG = OE$$

$$\therefore \frac{OG}{AO} = \frac{OE}{AO} \quad 1 \text{分}$$

$\because$ 四边形 $ABCD$ 是正方形

$$\therefore AB = BC, \angle ABC = \angle BCD = 90^\circ, AB \parallel CD$$

$$\therefore \frac{PC}{AB} = \frac{CG}{AG} = \frac{PC}{BC} \quad 1 \text{分}$$

$$\because \angle AHG = \angle ABC = 90^\circ$$

$$\therefore \angle FAB + \angle ABH = \angle CBP + \angle ABH = 90^\circ$$

$$\therefore \angle FAB = \angle CBP \quad 1 \text{分}$$

$\because AF$ 平分 $\angle CAB$

$$\therefore \angle FAC = \angle FAB$$

$$\therefore \angle FAC = \angle CBP \quad 1 \text{分}$$

$$\therefore \tan \angle FAC = \tan \angle CBP$$

$$\text{又} \because \angle AOE = \angle BCP = 90^\circ$$

$$\therefore \tan \angle FAC = \frac{OE}{OA}, \tan \angle CBP = \frac{PC}{BC}$$

$$\therefore \frac{OE}{OA} = \frac{PC}{BC} \quad 1 \text{分}$$

$$\therefore \frac{OG}{AO} = \frac{CG}{AG}$$

$$\therefore GO \cdot AG = CG \cdot AO \quad 1 \text{分}$$

24. (1) 解: $\because$ 抛物线的顶点为 $E(-1, 4)$

$\therefore$ 设抛物线的解析式为 $y = a(x+1)^2 + 4 (a \neq 0)$

2分

又 $\because$ 抛物线过点 $A(-3, 0)$

$$\therefore 4a + 4 = 0$$

$$a = -1$$

1分

$\therefore$ 这条抛物线的解析式为 $y = -(x+1)^2 + 4$

1分

(2) $\because A(-3, 0), E(-1, 4), C(0, 3)$

$\therefore$ 直线 AE 的解析式为 $y = 2x + 6$; 直线 AC 的解析式为 $y = x + 3$

$\because D$ 的横坐标为 $m, DK \perp x$ 轴

$$\therefore G(m, 2m+6)$$

1分

$$H(m, m+3)$$

1分

$$\because K(m, 0) \therefore GH = m+3, HK = m+3$$

1分

$$\therefore GH = HK$$

1分

(3) $\because C(0, 3), G(m, 2m+6), H(m, m+3)$

$$1^\circ \text{ 若 } CG = CH, \text{ 则 } \sqrt{m^2 + (2m+3)^2} = \sqrt{m^2 + m^2}$$

解得 $m_1 = -1, m_2 = -3$ 都是原方程的解, 但不合题意舍去.

所以这种情况不存在.

1分

$$2^\circ \text{ 若 } GC = GH, \text{ 则 } \sqrt{m^2 + (2m+3)^2} = m+3$$

解得 $m_1 = 0, m_2 = -\frac{3}{2}$ 都是原方程的解, 但 $m_1 = 0$ 不合题意, 舍去.

$$\therefore m = -\frac{3}{2}$$

1分

$$3^\circ \text{ 若 } HC = HG, \text{ 则 } \sqrt{m^2 + m^2} = m+3$$

$$\text{解得 } m = 3 - 3\sqrt{2}$$

1分

综上所述: 当 $\triangle CGH$ 是等腰三角形时, m 的值为 $-\frac{3}{2}$ 或 $3 - 3\sqrt{2}$.

1分

25. (1) 证明: $\because AD \perp BC, BH \perp AO$

$$\therefore \angle ADO = \angle BHO = 90^\circ$$

1分

在 $\triangle ADO$ 与 $\triangle BHO$ 中

$$\begin{cases} \angle ADO = \angle BHO \\ \angle AOD = \angle BOH \\ OA = OB \end{cases}$$

$$\therefore \triangle ADO \cong \triangle BHO$$

1分

$$\therefore OH = OD$$

1分

$$\text{又 } \because OA = OB$$

$$\therefore AH = BD$$

1分

(2) 联结 AB, AF

$\because AO$ 是半径, $AO \perp$ 弦 BF

$$\therefore \widehat{AB} = \widehat{AF} \therefore AB = AF \therefore \angle ABF = \angle AFB$$

1分

在 $\text{Rt}\triangle ADB$ 与 $\text{Rt}\triangle BHA$ 中 $\begin{cases} AH=BD \\ AB=BA \end{cases}$

$\therefore \text{Rt}\triangle ADB \cong \text{Rt}\triangle BHA \quad \therefore \angle ABF = \angle BAD$ 1 分

$\therefore \angle BAD = \angle AFB$

又 $\therefore \angle ABF = \angle EBA$

$\therefore \triangle BEA \sim \triangle BAF \quad \therefore \frac{BE}{BA} = \frac{BA}{BF} \quad \therefore BA^2 = BE \cdot BF$ 1 分

$\therefore y = BE \cdot BF \quad \therefore y = BA^2$

$\therefore \angle ADO = \angle ADB = 90^\circ \quad \therefore AD^2 = AO^2 - DO^2, AD^2 = AB^2 - BD^2$

$\therefore AO^2 - DO^2 = AB^2 - BD^2$ 1 分

$\therefore$ 直径 $BC = 8, BD = x$

$\therefore AB^2 = 8x$

$\therefore y = 8x$ 1 分

(3) 联结 OF

$\therefore \angle GFB$ 是公共角, $\angle FAE > \angle G$

$\therefore$ 当 $\triangle FAE \sim \triangle FBG$ 时, $\angle AEF = \angle G$ 1 分

$\therefore \angle BHA = \angle ADO = 90^\circ$

$\therefore \angle AEF + \angle DAO = 90^\circ, \angle AOD + \angle DAO = 90^\circ$

$\therefore \angle AEF = \angle AOD$

$\therefore \angle G = \angle AOD$ 1 分

$\therefore AG = AO = 4$

$\therefore \widehat{AB} = \widehat{AF} \quad \therefore \angle AOD = \angle AOF$

$\therefore \angle G = \angle AOF$, 又 $\therefore \angle GFO$ 是公共角

$\therefore \triangle FAO \sim \triangle FOG$ 1 分

$\therefore \frac{AF}{OF} = \frac{OF}{FG}$

$\therefore AB^2 = 8x, AB = AF \quad \therefore AF = 2\sqrt{2x}$

$\therefore \frac{2\sqrt{2x}}{4} = \frac{4}{4+2\sqrt{2x}} \quad$ 解得 $x = 3 \pm \sqrt{5}$ 是原方程的解 1 分

$\therefore 3 + \sqrt{5} > 4, \therefore$ 舍去

$\therefore BD = 3 - \sqrt{5}$ 1 分

浦东新区中考数学质量抽查试卷·参考答案

一、选择题:(本大题共 6 题,每题 4 分,满分 24 分)

1. B 2. C 3. A 4. A 5. C 6. B

二、填空题:(本大题共 12 题,每题 4 分,满分 48 分)

7. $\frac{2}{3}$ 8. $x < 3$ 9. $2(2+a)(2-a)$ 10. $-\bar{a} - \bar{b}$ 11. $x = -4$ 12. 3 13. 18 14. 45 15. 720

16. 1 或 5 17. 4 18. $\frac{35}{8}$

三、解答题:(本大题共 7 题,满分 78 分)

19. (本题满分 10 分)

$$\text{解:原式} = 2 \times \frac{\sqrt{2}}{2} - 1 + 2\sqrt{2} + 2 \quad (8 \text{ 分})$$

$$= 1 + 3\sqrt{2}. \quad (2 \text{ 分})$$

20. (本题满分 10 分)

$$\text{解方程: } \frac{x}{x+2} + \frac{x+2}{x-2} = \frac{8}{x^2-4}$$

$$\text{解:去分母得: } x(x-2) + (x+2)^2 = 8 \quad (4 \text{ 分})$$

$$\text{整理得: } x^2 + x - 2 = 0 \quad (2 \text{ 分})$$

$$\text{解得: } x_1 = 1, x_2 = -2 \quad (2 \text{ 分})$$

$$\text{经检验 } x_1 = 1 \text{ 是原方程的根, } x_2 = -2 \text{ 是原方程的增根} \quad (1 \text{ 分})$$

$$\text{原方程的根为 } x = 1 \quad (1 \text{ 分})$$

21. (本题满分为 10 分)

解:过点 O 作 $OD \perp AB$ 于 D

$$\text{在 Rt}\triangle AOC \text{ 中, } OA^2 + OC^2 = AC^2, AC = 5 \quad (2 \text{ 分})$$

$$\text{在 Rt}\triangle AOC \text{ 中, } \cos \angle OAC = \frac{OA}{AC} = \frac{4}{5}; \quad (2 \text{ 分})$$

$$\text{在 Rt}\triangle ADO \text{ 中, } \cos \angle OAD = \frac{DA}{AO}, \quad (2 \text{ 分})$$

$$\text{所以 } \frac{AD}{AO} = \frac{OA}{AC}, AD = \frac{16}{5}. \quad (1 \text{ 分})$$

因为在 $\odot O$ 中, $OD \perp AB$,

$$\text{所以 } AB = 2AD = 2 \times \frac{16}{5}, \quad (2 \text{ 分})$$

$$\text{所以 } AB = \frac{32}{5}. \quad (1 \text{ 分})$$

22. (本题满分 10 分,每小题 5 分)

解:(1) 设函数解析式为 $y = kx + b$, 将 $(0, 10)$ 、 $(40, 6)$ 分别代入 $y = kx + b$

$$\text{得} \begin{cases} 10 = b, \\ 6 = 40k + b. \end{cases} \quad (2 \text{ 分})$$

$$\text{解之得} \begin{cases} k = -\frac{1}{10}, \\ b = 10. \end{cases} \quad (1 \text{ 分})$$

$$\text{所以 } y = -\frac{1}{10}x + 10 (0 \leq x \leq 40) \quad (1+1 \text{ 分})$$

$$(2) \text{ 由 } \left(-\frac{1}{10}x + 11\right)x = 210 \quad (2 \text{ 分})$$

$$\text{解得 } x_1 = 30 \text{ 或 } x_2 = 70, \quad (1 \text{ 分})$$

$$\text{由于 } 0 \leq x \leq 40 \text{ 所以 } x = 30 \quad (1 \text{ 分})$$

$$\text{答:该产品的生产数量是 30 吨} \quad (1 \text{ 分})$$

23. (本题满分 12 分,第(1)、(2)小题各 6 分)

$$(1) \text{ 证明:因为,四边形 } ABCD \text{ 是平行四边形,所以, } \angle B = \angle D, \quad (2 \text{ 分})$$

因为 $\angle ECA = \angle D$, 所以 $\angle ECA = \angle B$, (2分)

因为 $\angle E = \angle E$,

所以 $\triangle ECA \sim \triangle ECB$ (2分)

(2) 解: 因为, 四边形 $ABCD$ 是平行四边形, 所以, $CD \parallel AB$, 即: $CD \parallel AE$

所以 $\frac{CD}{AE} = \frac{DF}{AF}$ (1分)

因为 $DF = AF$, 所以, $CD = AE$, (1分)

因为四边形 $ABCD$ 是平行四边形, 所以, $AB = CD$, 所以 $AE = AB$, 所以, $BE = 2AE$, (1分)

因为 $\triangle ECA \sim \triangle ECB$

所以 $\frac{AE}{CE} = \frac{CE}{BE} = \frac{AC}{BC}$ (1分)

所以 $CE^2 = AE \cdot BE = \frac{1}{2} BE^2$, 即 $\frac{CE}{BE} = \frac{\sqrt{2}}{2}$ (1分)

所以 $\frac{AC}{BC} = \frac{\sqrt{2}}{2}$. (1分)

24. (1) 将点 $B(3, 6)$ 代入解析式 $y = ax^2 - 4ax + 2$, 可得:

$6 = 9a - 12a + 2$, 解之得 $a = -\frac{4}{3}$. (2分)

所以二次函数解析式为 $y = -\frac{4}{3}x^2 + \frac{16}{3}x + 2$. (1分)

点 A 的坐标为 $(0, 2)$. (1分)

(2) 由题意, $C(1, 6)$, $BC = 2$, $AB = 5$, $\tan \angle CBA = \frac{4}{3}$. (1分)

过点 C 作 $CH \perp AB$ 于点 H . $\therefore CH = \frac{8}{5}$, $BH = \frac{6}{5}$, $AH = \frac{19}{5}$ (2分)

$\therefore \tan \angle CAB = \frac{8}{19}$. (1分)

(3) 由题意, $AB = AB_1 = 5$, 从而点 B_1 的坐标为 $(0, -3)$ 或 $(0, 7)$. (2分)

① 若点 $B_1(0, -3)$, 设 $P(x, 0)$, 由 $PB = PB_1$, 有 $(x-3)^2 + 6^2 = x^2 + 3^2$,
解得: $x = 6$, 即 $P(6, 0)$ (1分)

② 若点 $B_1(0, 7)$, 设 $P(x, 0)$, 由 $PB = PB_1$, 有 $(x-3)^2 + 6^2 = x^2 + 7^2$,
解得: $x = -\frac{2}{3}$, 即 $P(-\frac{2}{3}, 0)$ (1分)

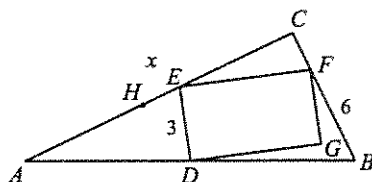
综合知, 点 P 的坐标为 $(6, 0)$ 或 $(-\frac{2}{3}, 0)$.

25. (1) 如图, $\because AD = \frac{1}{2}AB = 5 \therefore DE = FG = 5 \times \frac{3}{4} = \frac{15}{4}$.

(2分)

$BG = \frac{3}{4}FG = \frac{3}{4} \times \frac{15}{4} = \frac{45}{16}$

$\therefore DG = 5 - \frac{45}{16} = \frac{35}{16}$. 即 $DE = \frac{15}{4}$, $EF = \frac{35}{16}$. (2分)



(2) 过点 D 作 $DH \perp AC$ 于点 H , 从而 $DH = 3$. 易得 $\triangle DHE \sim \triangle ECF$, 由 $\frac{DE}{EF} = \frac{1}{2}$, 可得 $EC = 2DH$

$$=6, EH = \frac{1}{2}x - 6. \quad (3 \text{ 分})$$

$$\text{所以 } DE^2 = 3^2 + \left(\frac{x}{2} - 6\right)^2 = \frac{x^2}{4} - 6x + 45. \quad (1 \text{ 分})$$

$$\therefore y = DE \cdot EF = 2DE^2 = \frac{x^2}{2} - 12x + 90. \quad (1 \text{ 分})$$

(3) 由题意, 点 G 可以在边 BC 或者 AB 上.

① 如左图若点 G 在边 BC 上, 从而由 $DE = 3$, 可知 $EF = \frac{9}{2}$, 于是 $AC = 2EF = 9$; (2 分)

② 如右图, 若点 G 在边 AB 上. 记 $AD = DB = a$, 矩形边长 $DE = 2b$, $EF = 3b$, 由 $\triangle ADE \sim \triangle FGB$, 可得 $\frac{AD}{DE} = \frac{DG}{GB}$, 即 $\frac{a}{2b} = \frac{2b}{a-3b}$, 化简可得 $a^2 - 3ab - 4b^2 = 0$, 因式分解后有: $a = 4b$, 即 $AD = 2DE$. 而由 $\triangle ADE \sim \triangle ACB$, 所以 $AC = 2BC$, 从而 $AC = 12$. (3 分)

综合知, AC 的值为 9 或 12.